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EVALUATION OF PERIODIC PROCESSES WITH TWO MODULATED INPUTS BASED ON NONLINEAR FREQUENCY RESPONSE ANALYSIS

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Motivation

Deliberate periodic operations

- One aspect of Process Intensification
- The process performance can be enhanced by periodic modulation of some inputs around a chosen steady-state
- This enchantment (Δ) is a result of the system nonlinearity
- Only in some cases the periodic operation is superior to the steady-state
- The periodic steady state – the quasi-stationary response of the system ($t\to\infty$) – in control theory known as **frequency response**
- For a nonlinear system defined by a set of Frequency Response Functions (FRFs) of the first, second, third, … order

The Aim

To develop a method, based on the **frequency response** theory, to calculate the periodic steady state **directly and analytically**, without numerical integration

Theoretical Basis

Frequency response of a weakly nonlinear system with one modulated input

$$x = x_s + A\cos(\omega t) \xrightarrow[t\to\infty]{} y = y_s + y_{DC} + B_I\cos(\omega t + \varphi_I) + B_{II}\cos(2\omega t + \varphi_{II}) + \cdots$$

y_{DC} – *nonperiodic term* – responsible for average performance of the periodic process – defines the process improvement through periodic operation (Δ)

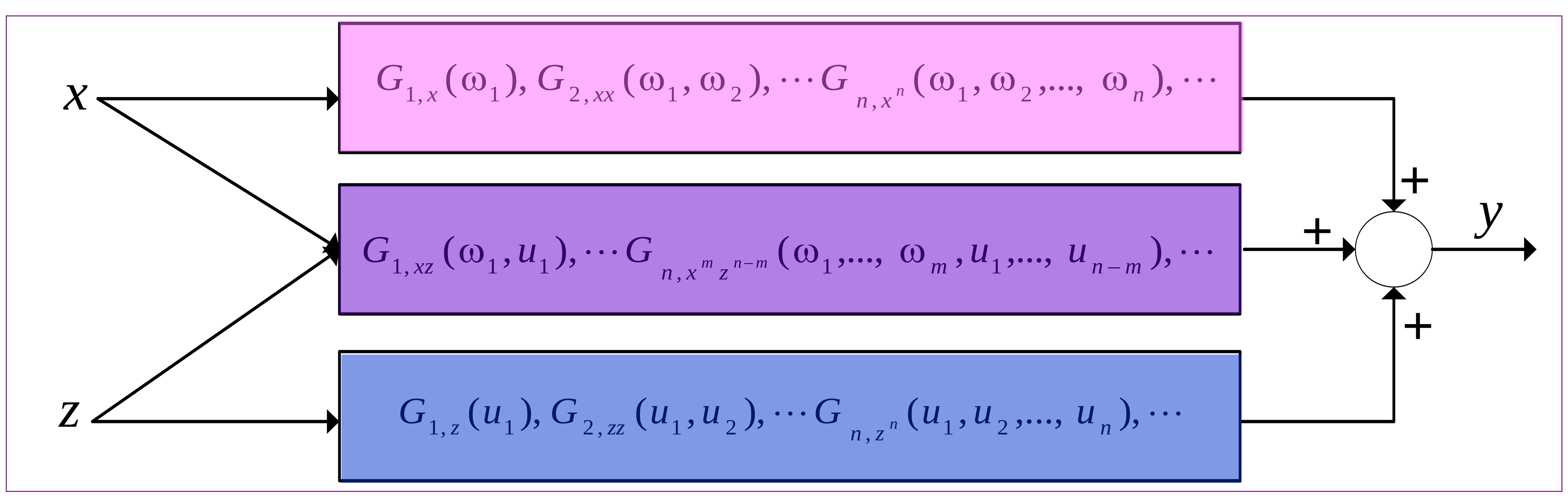
$$\Delta \equiv y_{DC} = 2(A/2)^2 G_2(\omega,-\omega) + 6(A/2)^4 G_4(\omega,\omega,-\omega,-\omega) + \cdots$$

The dominant term of y_{DC} proportional to $G_2(\omega,-\omega)$ – the asymmetrical second order FRF

$$y_{DC} \approx 2(A/2)^2 G_2(\omega,-\omega)$$

$G_2(\omega,-\omega)$ determines the sign and the approximate value of Δ

Extension to Periodic Processes with Two Modulated Inputs



$$G_{1,x}(\omega_1),\ G_{2,xx}(\omega_1,\omega_2),\cdots G_{n,x^n}(\omega_1,\omega_2,\ldots,\omega_n),\cdots$$

$$G_{1,xz}(\omega_1,u_1),\cdots G_{n,x^m z^{n-m}}(\omega_1,\ldots,\omega_m,u_1,\ldots,u_{n-m}),\cdots$$

$$G_{1,z}(u_1),\ G_{2,zz}(u_1,u_2),\cdots G_{n,z^n}(u_1,u_2,\ldots,u_n),\cdots$$

$$y_{DC} \approx 2\left(\frac{A}{2}\right)^2 G_{2,xx}(\omega,-\omega) + 2\left(\frac{B}{2}\right)^2 G_{2,zz}(\omega,-\omega) + \left(\frac{A}{2}\right)\left(\frac{B}{2}\right)\cos(\varphi)\left(G_{2,xz}(\omega,-\omega) + G_{2,xz}(-\omega,\omega)\right)$$

$$\left.\begin{aligned} x &= A\cos(\omega t) \\ z &= B\cos(\omega t + \varphi) \end{aligned}\right\} \Rightarrow y_{DC} = y_{DC,x} + y_{DC,z} + y_{DC,xz}$$

$$y_{DC,x} = 2\left(\frac{A}{2}\right)^2 G_{2,xx}(\omega,-\omega) + \cdots, \quad y_{DC,z} = 2\left(\frac{B}{2}\right)^2 G_{2,zz}(\omega,-\omega) + \cdots$$

$$y_{DC,xz} = \left(\frac{A}{2}\right)\left(\frac{B}{2}\right)\cos(\varphi)\left(G_{2,xz}(\omega,-\omega) + G_{2,xz}(-\omega,\omega)\right) + \ldots$$

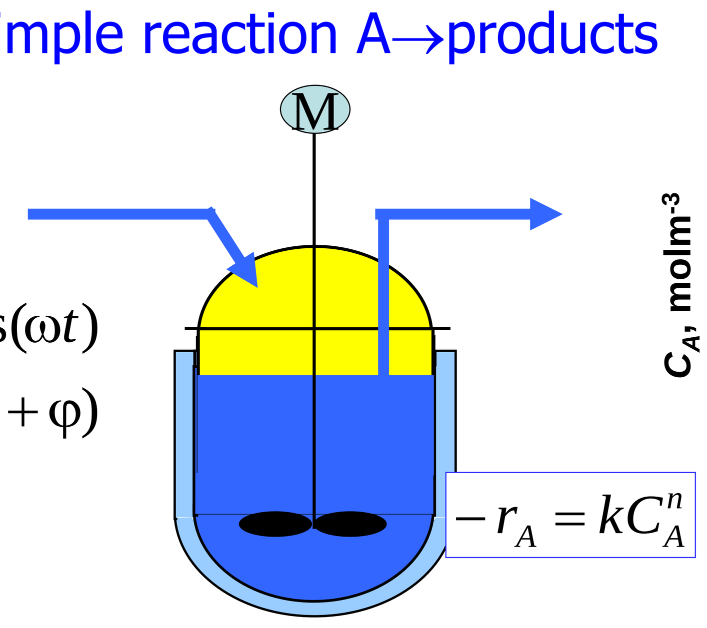
The sign of y_{DC} determined by $G_{2,xx}(\omega,-\omega)$, $G_{2,zz}(\omega,-\omega)$, $G_{2,xz}(\omega,-\omega)$ and $\cos(\varphi)$

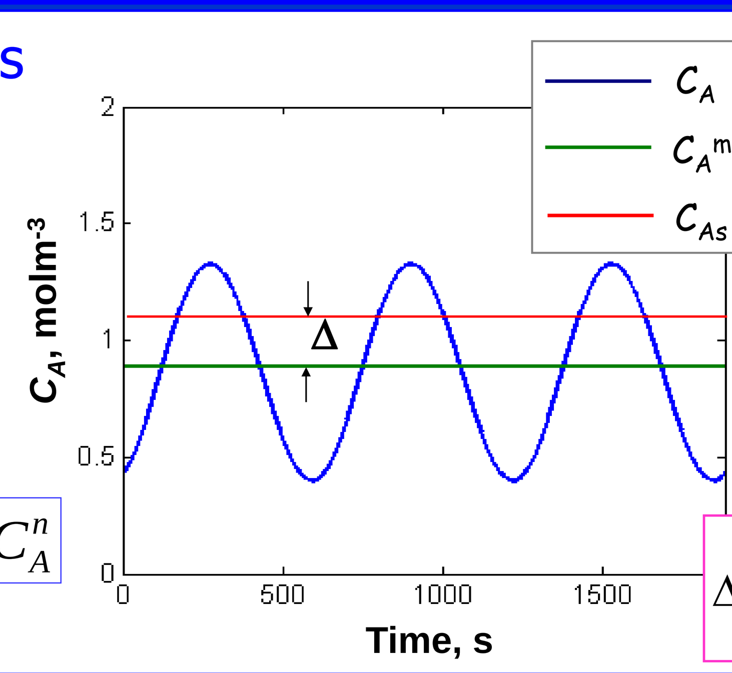
In phase modulation of the inputs: $\cos(\varphi)=1$
Out of phase modulation of the inputs: $\cos(\varphi)=-1$

Example – CSTR with Modulation of Inlet Concentration and Flow-rate

Simple reaction A→products

$$C_{Ai}(t) = C_{Ai,s} + A\cos(\omega t)$$
$$F(t) = F_s + B\cos(\omega t + \varphi)$$



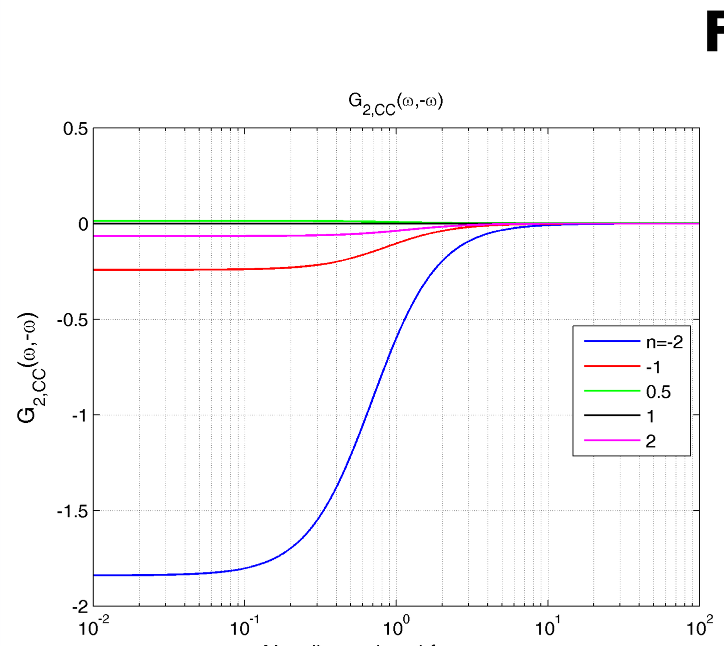


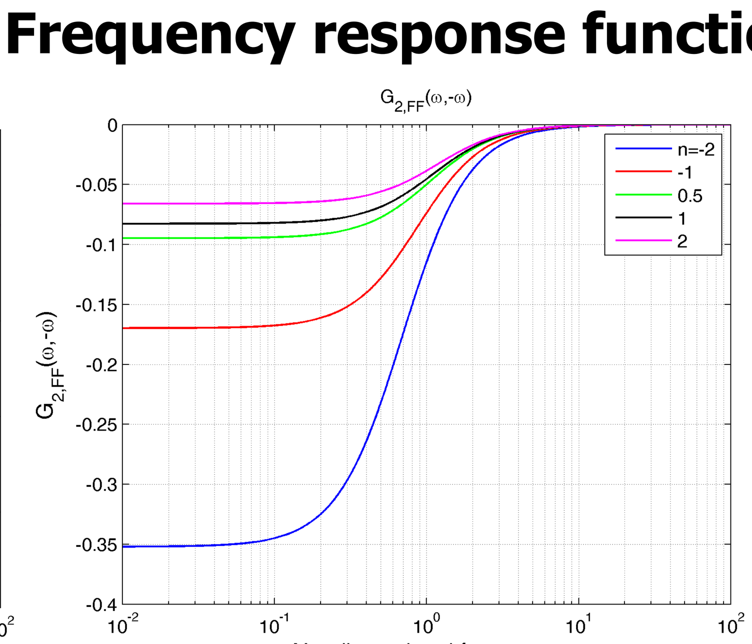
$c_{A,s}$ – steady-state outlet concentration of the reactant A (steady-state operation)
c_A^m – mean outlet concentration of the reactant A, during periodic operation
Δ – the difference:

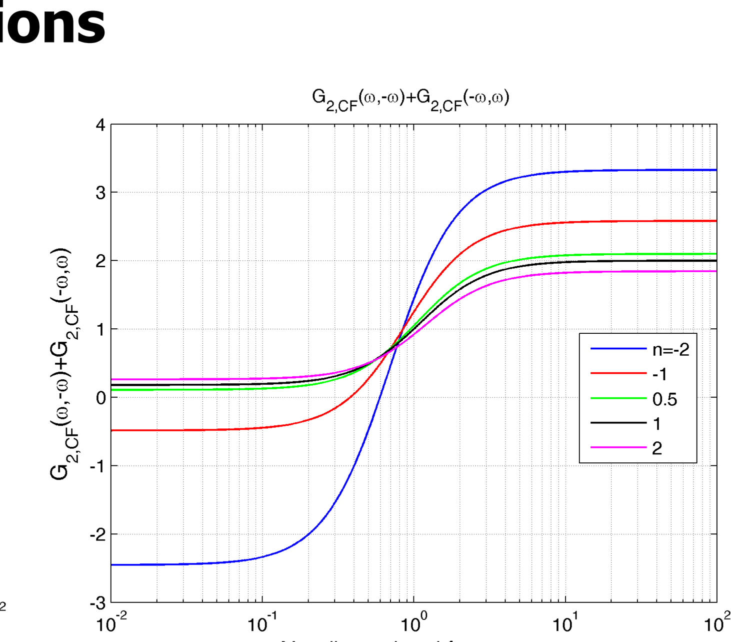
- $\Delta<0$ – periodic operation is superior to steady-state operation
- $\Delta=0$ – periodic operation has no influence
- $\Delta>0$ – periodic operation is worse than steady-state operation

$$\Delta \equiv y_{DC} \approx 2\left(\frac{A}{2}\right)^2 G_{2,CC}(\omega,-\omega) + 2\left(\frac{B}{2}\right)^2 G_{2,FF}(\omega,-\omega) + \left(\frac{A}{2}\right)\left(\frac{B}{2}\right)\cos(\varphi)\left(G_{2,CF}(\omega,-\omega) + G_{2,CF}(-\omega,\omega)\right)$$

Frequency response functions

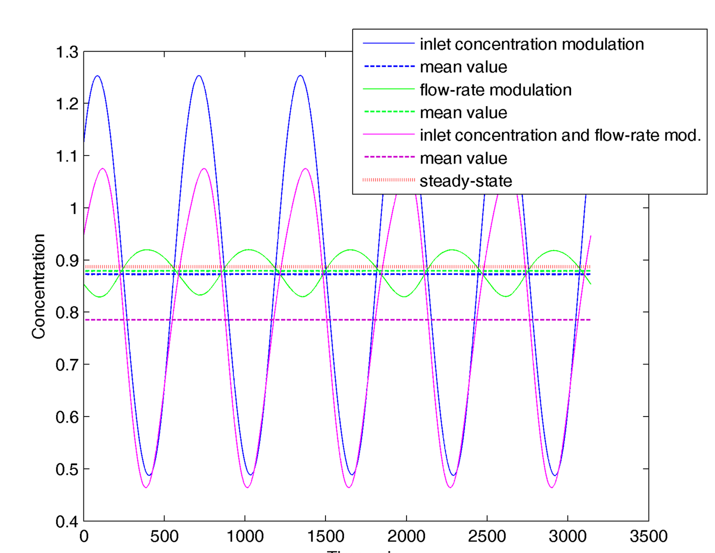






Example: $C_{Ai,s}$=1 mol/m³, k=0.001, V/F=100 s)

Comparison with numerical solution



Modulation	Δ (numer.)	y_{DC} (FR)
C_{Ai}	-0.0144	-0.0116
F	-0.0081	-0.0081
C_{Ai} and F In-phase	+0.0578	+0.0498
C_{Ai} and F Out-of-phase	-0.1022	-0.0894

Example: n=-1, ω=0.01 rad/s, A=0.5, B=0.5

Conclusions

- Evaluation of periodic processes with two modulated inputs, using nonlinear frequency response analysis possible
- Three asymmetrical second order FRFs necessary: two for separate inputs and one mixed
- Extension to periodic processes with three and more modualtedinputs possible and streigtforward