Heat transfer by liquid convection in particulate fluidized beds

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Abstract: In this work the theoretical model for heat transfer from a wall to a liquid-solid fluidized bed by liquid convective mechanism has been proposed and developed. The model is based on thickness of boundary layer and film theory. The key parameter in the model is the distance between two adjacent particles which collide with the wall. According to the proposed model, the liquid convective heat transfer in a fluidized bed is 4 to 5 times more intense than in a single-phase flow. Additionally, the wall-to-bed heat transfer coefficient has been measured experimentally in water–glass particles fluidized bed, for different particle sizes. Comparison of the model prediction with experimental data has shown that size of the particles strongly influences the mechanism of heat transfer. For fine particles of 0.8 mm in diameter, the liquid convective heat transfer model represents adequately the experimental data, indicating that particle convective mechanism is negligible. For coarse particles of 1.5–2 mm in diameter, the liquid convective heat transfer mechanism accounts for 60 % of the overall heat transfer coefficient.

Keywords: model; boundary layer; film theory.

INTRODUCTION

Liquid–solid fluidized beds are characterized by high heat and mass transfer rates, uniform temperature, and ability to reduce fouling. These specific features give rise to their numerous traditional and novel applications as: reactors,[1] bioreactors,[2–4] in food processing,[5] for waste water treatment[6] and thermal energy generation.[7]

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Heat transfer in liquid-solid fluidized beds is a complex phenomenon, affected by physical properties of liquid and solid, operational parameters and geometry factors. Wall-to-bed heat transfer coefficient is commonly interpreted as a sum of contributions of liquid convective (α_{lc}) and particle convective heat transfer mechanisms (α_{pc}). The major resistance to heat transfer is considered to be in the laminar sublayer at the wall, where forced liquid convection takes place. Particle convective contribution refers to the heat transferred by particles due to their collisions with the wall, and the movement in and out of the thermal boundary layer. The key factors affecting the heat transfer rate are thickness of boundary layer (liquid convective mechanism) and the particle velocities, which determine the frequency of contacts between particles and wall (particle convective mechanism). Both the thickness of boundary layer and the particle velocities are generally difficult to quantify, and that can explain the fact that there is a very few theoretical studies on mechanism of heat transfer in liquid-solid fluidized in literature.[8,9]

Heat transfer in liquid–solid fluidized beds has been much more investigated experimentally,[10–15] yielding in empirical correlations for the prediction of heat transfer coefficient, which are limited to the certain ranges of operating conditions. The typical forms of empirical correlations have been suggested by Mouroyama *et al.*:[11]

$$\mathrm{Nu}_{\mathrm{p}} = \frac{0.137}{\varepsilon} \left(1 - \varepsilon\right)^{0.271} \mathrm{Re}_p^{0.729}\, \mathrm{Pr}^{1/3} \tag{1}$$

and Haid *et al.*:[13]

$$\mathrm{Nu}_{\mathrm{p}} = 0.1493\, \mathrm{Re}_p^{0.729}\, \mathrm{Pr}^{0.52} \left(\frac{\rho_{\mathrm{p}} - \rho}{\rho}\right)^{0.03} \left(\frac{d_{\mathrm{p}}}{D}\right) \varepsilon^{-1.41} \left(1 - \varepsilon\right)^{0.271} \tag{2}$$

Jamialahmadi *et al.*[16] have compared the predictions for heat transfer coefficient from a number of empirical correlations, and concluded that there are quite big discrepancies among them. Therefore, the same authors attempted to generalize the correlation for heat transfer coefficient. They suggested an approach where the total heat transfer area (A) is divided into the part where the heat is transferred by liquid convection (A_c) and the part where the particle convective mechanism takes place (A_p). Each of the mechanisms contribute to the overall heat transfer coefficient (α) proportionally to the fraction of area where it occurs, as it is given by:

$$\alpha = \alpha_{\mathrm{lc}} + \alpha_{\mathrm{pc}} = \left(1 - \frac{A_{\mathrm{p}}}{A}\right)\alpha_{\mathrm{c}} + \frac{A_{\mathrm{p}}}{A}\alpha_{\mathrm{p}} \tag{3a}$$

They assumed the local liquid convective heat transfer coefficient (α_c) to be as in the single phase flow, defined by equation of Gnielinski *et al.*[17] and the particle convective heat transfer coefficient (α_p) is represented by unsteady-state

heat transfer defined by equation of Mickley and Faribacks.[18] The fraction of area affected by particles (A_p/A) has been defined by an empirical Eq. (3b), as the dependence on particle concentration ($1-\varepsilon$) and particle to column diameter ratio (d_p/D):

$$\frac{A_p}{A} = 1.5\left(\frac{d_p}{D}\right)^{0.256}\left(1-\varepsilon\right)^{0.507} \tag{3b}$$

The exponents in Eq. (3b) have been determined by fitting the large number of experimental data for the heat transfer coefficient, measured by various authors, to the Eq. (3a).

Khan and Elkamel[9] have proposed the theoretical model for heat transfer coefficient between immersed flat element and fluidized bed. That model includes the energy balance equation in order to assess the fluid convective contribution and Fourier equations for the unsteady-state thermal conduction in order to assess the particle conductive and fluid conductive components of the heat transfer coefficient. The necessity of numerical solutions of the rigorous model equations restricts the practical applicability of this model.

Wasmund and Smith[8] have developed the model for the contribution of the particle convective heat transfer mechanism, based on the particle velocities and radial temperature profiles. By comparison of the modelled particle convective contribution, with the experimental data for overall heat transfer coefficient, they concluded that the particle convective mechanism accounts for 30–60 % of the overall heat transfer coefficient, depending on the ratio of particle to fluid thermal resistance.

The aim of this work was a theoretical study of liquid convective heat transfer mechanism in liquid–solid fluidized bed. A new model, which predicts the contribution of liquid convective mechanism, has been developed based on the boundary layer thickness and film theory. For comparison, the wall-to-bed heat transfer coefficients have been measured experimentally for different particle sizes.

Theoretical model for liquid convective heat transfer

Some of the early theories of heat transfer in fluidized beds are based on the existence of the fluid boundary layer adjacent to the column wall.[19,20] The developed theoretical models[8,9,16] are commonly based on the fact that the main resistance to heat transfer lies in the fluid film next to the column wall, while the resistance of the bed is assuming to be negligible. The fluidizing particles influence the heat transfer in two ways: a) their motion affects the thickness of boundary layer, a result of which is the liquid convective heat transfer and b) they transfer heat themselves by absorbing it on the hot wall and releasing it in the bulk of the bed.

The model for liquid convective heat transfer from the wall, which will be presented in this work, is based on the concept that the presence of fluidizing particles has two effects: a) it reduces the thickness of the liquid boundary layer compared to the single-phase flow and b) the free area for heat transfer by liquid convection is reduced, since a certain part of the area is occupied by particles which collide with it.

In the single-phase flow of liquid over a flat surface, the continuous hydrodynamic boundary layer is formed. Under laminar conditions ($Re_x < 2\times10^5$), the local thickness of the boundary layer at is defined by:[21]

$$\delta_{Hx} = 4.64x Re_x^{-1/2} \tag{4}$$

where Re_x is the local Reynolds number at a distance x from the leading edge. An average thickness over a certain length L is defined by:

$$\delta_H = \frac{1}{L}\int_0^L \delta_{Hx}dx = 3.09L\, Re_L^{-1/2} \tag{5}$$

When the heat transfer between the surface and fluid takes place the thermal boundary layer has been formed parallelly with hydrodynamic boundary layer. If Prandtl number (Pr) is larger than unity, the thermal boundary layer is contained within the hydrodynamic one and the thicknesses of those two layers are related by:[21]

$$\frac{\delta_T}{\delta_H} = Pr^{-1/3} \tag{6}$$

Considering Eq. (5), the thickness of the thermal boundary layer can be expressed by:

$$\delta_T = 3.09L Re_L^{-1/2} Pr^{-1/3} \tag{7}$$

According to the film theory, the heat transfer coefficient can be defined as the ratio of fluid thermal conductivity and the thickness of the thermal boundary layer as it is given by:

$$\alpha_{sf} = \frac{\lambda_f}{\delta_T} = 0.324\frac{\lambda_f}{L} Re_L^{1/2} Pr^{1/3} \tag{8}$$

Eq. (8) gives the value of the convective heat transfer coefficient in a single-phase flow (α_{sf}). In the case of fluidized bed, the moving particles collide with the heat transfer surface (column wall) and cause brakeage of the continuous boundary layer. We assume that the thermal boundary layer is formed on the distance between two adjacent particles which collide with the wall, as it is depicted in Fig. 1a. This distance has been approximated by the mean distance between collisions according to the kinetic theory of gases. This approximation is based on the study of Carlos and Richardson,[22] who have shown that the movement of the particles in the particulate fluidized bed can be satisfactorily represented by the movement of molecules in the kinetic theory of gases. Consequently, the

authors define the mean distance between collisions by Eq. (9), analogously to a mean free path of molecules:[22]

$$L = \frac{d_p}{6\sqrt{2}\left(1-\varepsilon\right)} \tag{9}$$

By incorporating Eq. (9) into the Eq. (7), the thickness of thermal boundary layer in the fluidized bed is defined by:

$$\delta_{Tfb} = 1.06 \frac{d_p}{\sqrt{\left(1-\varepsilon\right)}} Re_p^{-1/2} Pr^{-1/3} \tag{10}$$

In Eq. (10) Re_p is the particle Reynolds number. The local liquid convective heat transfer coefficient in a fluidized bed (α_{sf}) is then defined by:

$$\alpha_{fb} = \frac{\lambda_f}{\delta_{Tfb}} = 0.943 \frac{\lambda_f}{d_p} \sqrt{\left(1-\varepsilon\right)} Re_p^{1/2} Pr^{1/3} \tag{11}$$
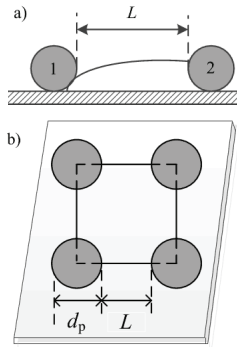

Fig. 1. a) Distance between adjacent particles which collide with the wall and b) segment of the wall representing the coverage of the area by colliding particles.

Similarly to the approach of Jamialahmadi et al.,[16] we assumed that the liquid convective mechanism occurs at the part of the heat transfer area which is not occupied by particles. In order to estimate the fraction of free area, we would use the simplified representation based on the square setup of particles which collide with the wall as it is depicted in Fig. 1b. The total area of the square unit is $(d_p+L)^2$ and the area covered by particles is $(d_p^2\pi)/4$. The fraction of free area (θ_{free}) can be calculated using:

$$\theta_{free} = \frac{\left(d_p + L\right)^2 - \dfrac{d_p^2 \pi}{4}}{\left(d_p + L\right)^2} \tag{12}$$

The contribution of liquid convective mechanism to the heat transfer coefficient in a fluidized bed is defined by:

$$\alpha_{lc} = \theta_{free}\alpha_{fb} \tag{13}$$

EXPERIMENTAL

The heat transfer coefficients have been measured in the experimental apparatus schematically shown in Fig. 2. The fluidization column was the copper pipe equipped with the heating jacket. The diameter of the inner pipe was $D = 25.4$ mm and the length of heating section was $H = 0.7$ m. The fluidizing liquid was water, which was at the room temperature at inlet, and the heating medium was saturated water vapour, flowing in a counter current mode. The experiments have been performed at atmospheric pressure. Fluidizing particles of different sizes were made of glass and their characteristics are given in Table I. According to the diagram of the hydrodynamic behaviour of fluidized systems, given by Jamialahmadi *et al.*,[16] glass particles smaller than 2.2 mm fluidized by water exhibit particulate behaviour, which is the case for all particles used in this study.
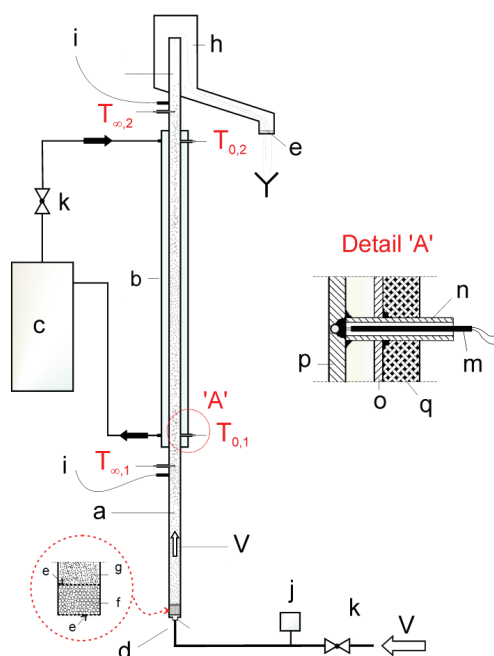

Fig. 2. Experimental apparatus (a – column, b – heating section, c – steam generator, d – inlet nozzle, e – screen, f – distributor, g – fluidized bed, h – overflow, i – pressure taps, j – flowmeter, k – valve, m – Ni–Cr thermocouple, n – cooper tube 8/6 mm, o – jacket wall, p – column wall, q – thermoisolation).

TABLE I. Characteristics of the particles and range of superficial liquid velocity

d_p / mm	ρ_p / kg m⁻³	$U_{t\infty}$ / m s⁻¹	U / m s⁻¹	Re_p
0.8	2923	0.148	0.04–0.139	60–160
1.20	2641	0.188	0.05–0.206	100–280
1.52	2710	0.260	0.06–0.209	200–400
1.94	2507	0.288	0.06–0.28	265–620

After establishing the desired water flowrate, the following quantities have been measured in a steady-state: wall temperature at inlet and outlet ($T_{0,1}$ and $T_{0,2}$), water temperature at inlet and outlet ($T_{\infty,1}$ and $T_{\infty,2}$) and pressure drop over the heating section (ΔP). The range of the tested superficial liquid velocities for each particle diameter is given in Table I, together with the corresponding particle Reynolds number. The bed voidage corresponding to each

superficial velocity is calculated based on the pressure drop using the basic equation of fluidization:

$$\varepsilon = 1 - \frac{\Delta P}{(\rho_p - \rho) g H} \qquad (14)$$

The wall-to-bed heat transfer coefficient has been calculated based on the heat balance for heating section:

$$\alpha = \frac{G_f c_{pf} (T_{\infty,2} - T_{\infty,1})}{D \pi H \Delta T_{ln}} \qquad (15)$$

In Eq. (15) G_f is the mass flow rate of water, c_{pf} is the specific heat capacity of water and the logarithmic mean temperature difference is defined by:

$$\Delta T_{ln} = \frac{(T_{0,2} - T_{\infty,2}) - (T_{0,1} - T_{\infty,1})}{\ln((T_{0,2} - T_{\infty,2}) / (T_{0,1} - T_{\infty,1}))} \qquad (16)$$

RESULTS AND DISCUSSION

As it can be concluded from empirical correlations as well as the proposed model (Eq. (13)), the bed voidage has a key influence on the heat transfer in fluidized bed. Therefore, it is important to have the appropriate correlation for prediction of the dependence of the bed voidage on superficial velocity. Firstly, we attempted to correlate the experimental data for bed voidage with classical Richardson-Zaki correlation[23] (Eq. (17)), based on free particle terminal velocity ($U_{t\infty}$, Table I):

$$\varepsilon = \left(\frac{U}{U_{t\infty}}\right)^{1/n} \qquad (17)$$

It turned out that Eq. (17) fits the experimental data only for particles $d_p = 0.8$ mm (Fig. 3). For all larger particles Eq. (17) underestimates experimental data, which means that the effect of the column wall reduces the particle terminal velocity. This effect has been taken into account by correction factor K (Eq. (18)), suggested by Khan and Richardson.[24] The modified Richardson–Zaki equation (Eq. (19)) fits adequately the experimental bed voidage for particles of 1.1, 1.52 and 1.97 mm (only the data for $d_p = 1.97$ mm has been shown on Fig. 3):

$$K = 1 - 1.15\left(\frac{d_p}{D}\right)^{0.6} \qquad (18)$$

$$\varepsilon = \left(\frac{U}{K U_{t\infty}}\right)^{1/n} \qquad (19)$$

Fig. 4 shows the experimental heat transfer coefficient *vs.* bed voidage for all four particle sizes. Similar to the findings of other authors, the heat transfer coefficient rises with the increase of particle size in the given rage of particle diameters. The difference is more pronounced for lower bed voidage, while for

the loose beds ($\varepsilon > 0.9$) the size of particles has no influence on heat transfer coefficient.
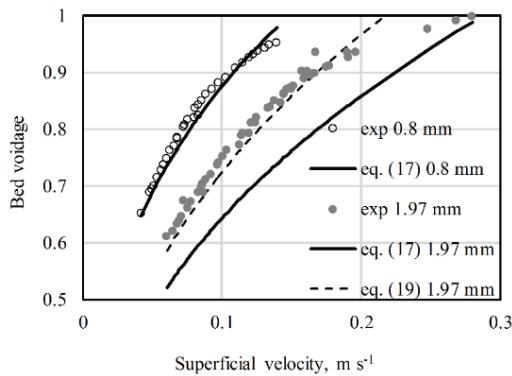

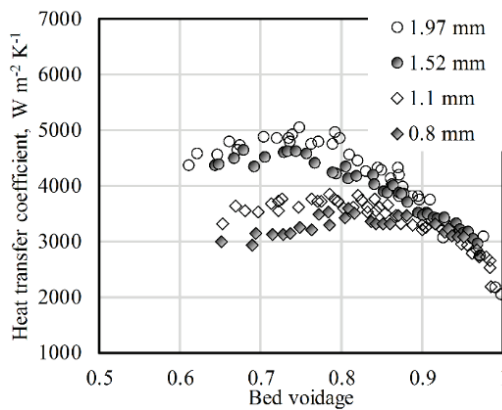Fig. 3. Bed voidage *vs.* superficial velocity.

Fig. 4. Experimental heat transfer coefficient *vs.* bed voidage.

It can be also observed from Fig. 4, that the heat transfer coefficient exhibits a slight maximum for the bed voidage of about 0.74 for coarser particles, while it is somewhat shifted towards higher bed voidage (about 0.8) for smaller particles. This maximum has been usually interpreted in literature[9] as a consequence of the opposed effects of increase in the liquid velocity and particle velocity on one side and decrease in particle concentration on the other side.

The comparison of the experimental data for heat transfer coefficient with predictions of literature correlations defined by Eqs. (1), (2) and (3a) has been shown in Fig. 5a for $d_p = 0.8$ mm and in Fig. 5b for $d_p = 1.52$ mm. It can be seen that the correlation of Muroyama *et al.*[11] (Eq. (1)) fits very well the experimental data for $d_p = 1.52$ mm (mean relative deviation 3.5 %, Fig. 5b), while all three correlations overpredict the experimental data for $d_p = 0.8$ mm (Fig. 5a).

Fig. 6 shows the liquid convective heat transfer contribution (α_{lc}) *vs.* bed voidage, calculated from the model Eq. (13), for different particle sizes. The

liquid convective heat transfer coefficient for the single-phase flow (α_{sf}), calculated from Eq. (8), using the pipe diameter $D = 25.4$ mm as characteristic geometry, is given for comparison. It should be noted that bed voidage at abscissa corresponds to superficial velocity according to Eq. (17).
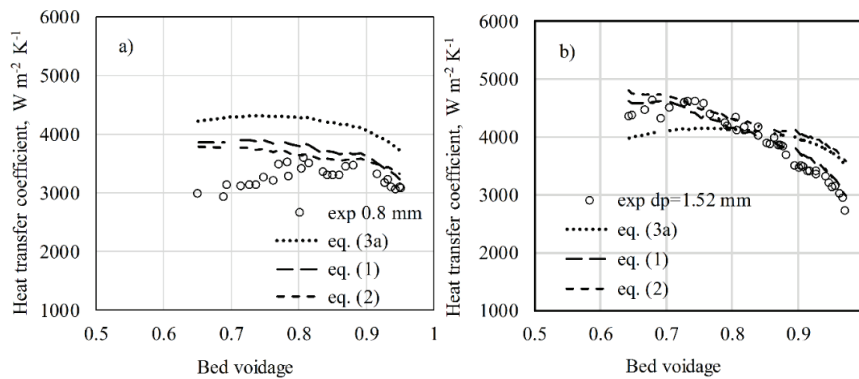

Fig. 5. Comparison of experimental data with literature correlations for d_p:
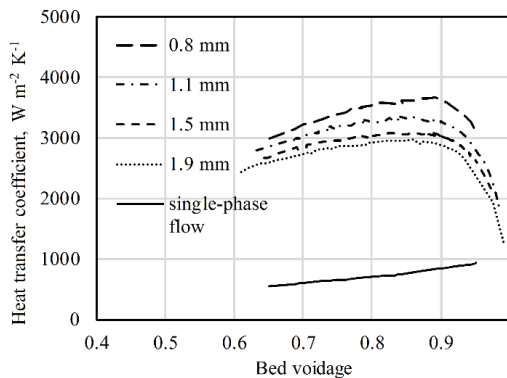a) 0.8 and b) 1.52 mm.

Fig. 6. Model prediction for liquid convective heat transfer contribution in fluidized and single-phase flow.

The predicted values of α_{lc} in the fluidized bed are 4–5 times higher compared to the single-phase flow. The values of α_{lc} in the fluidized bed exhibit slight increase with bed voidage up to the maximum at $\varepsilon \approx 0.87$ followed by a sharp decrease. Since α_{lc} depends on δ_T and θ_{free}, in order to explain the trend α_{lc} vs. ε, the dependence of δ_T and θ_{free} on superficial velocity (i.e., bed voidage) will be analysed. The equation for the thickness of the thermal boundary layer (Eq. (7)) can be written in the form:

$$\delta_T \propto \left(\frac{L}{U} \right)^{1/2} \tag{20}$$

The increase in superficial velocity directly influences δ_T such that it decreases it, as in the single-phase flow. However, in the fluidized bed, the increase in velocity simultaneously increases the distance between two collisions, L (Fig. 7), which increases the thickness of the boundary layer (Eq. (19)). Those two opposed effects lead to the nearly constant value of δ_T in fluidized bed (δ_{Tfb}) up to $\varepsilon \approx 0.87$, as it is presented in Fig. 7. For $\varepsilon > 0.87$, the sharp increase in L predominates the increase in the velocity, causing the increase of δ_{Tfb}. The fraction of free area (θ_{free}) increases with bed voidage as it is presented in Fig. 8. Regarding the trend of α_{lc} *vs.* ε (Fig. 6), it can be concluded that the increase in α_{lc} for $\varepsilon < 0.87$ is due to the increase in θ_{free}, since $\delta_{Tfb} \approx$ const. The sharp decrease in α_{lc} for $\varepsilon > 0.87$ is due to the increase in δ_{Tfb}. It can be seen from Fig. 6 that the size of the particles does not have a strong influence on α_{lc}. However, the higher values of α_{lc} are obtained for smaller particle diameter. That is the consequence of lower L for the smaller d_p for the same bed voidage (Eq. (9)).
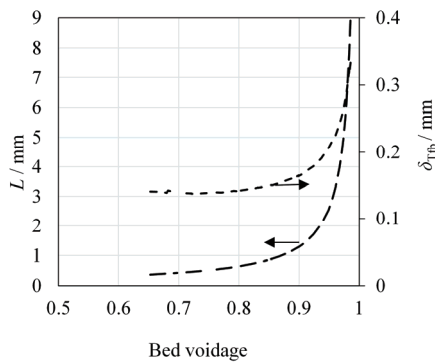

Fig. 7. Distance between two consecutive collisions and thickness of thermal boundary layer in a fluidized bed *vs.* bed voidage ($d_p = 1.1$ mm).

Fig. 9 shows the comparison of the modelled liquid convective component (Eq. (13)) with the experimental data for overall heat transfer coefficient for each particle diameter. As expected, the curves which represent the liquid convective component (α_{lc}) lie below the experimental data for d_p 1.1, 1.52 and 1.97 mm (Fig. 9b–d). The difference between the experimental data and model prediction for α_{lc} corresponds to the contribution of particle convective heat transfer (α_{pc}), which is larger for larger particle diameter and accounts for 17 % for $d_p = 1.1$ mm, 40 % for $d_p = 1.52$ mm and 43 % for $d_p = 1.97$ mm. This is logical since in the bed of larger particles higher liquid velocity is needed to achieve the same bed voidage. Higher liquid velocity cause higher particle velocity and consequently higher frequency of contacts between particles and the wall. This is in agreement with the findings of Wasmund and Smith.[8] According to their model for the system water – 1.53 mm glass particles, the particle convective contribution accounts for 50 % of the overall heat transfer coefficient, which is close to our result of 40 % for the same system.
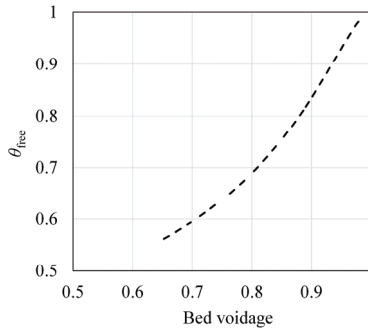
Fig. 8. Fraction of heat transfer area free for liquid flow *vs.* bed voidage (d_p= 1.1 mm).
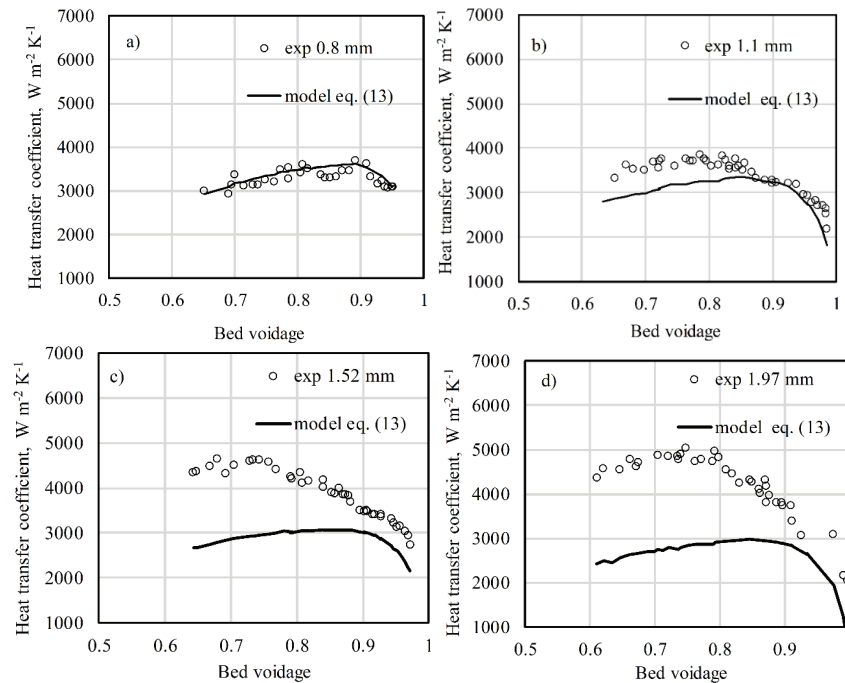

Fig. 9. Comparison of the experimental data for heat transfer coefficient with modelled liquid convective contribution for d_p: a) 0.8, b) 1.1, c) 1.52 and d) 1.97 mm.

One very interesting result has been obtained for the smallest particles $d_p =$ = 0.8 mm where the model for liquid convective contribution adequately represents the experimental data (Fig. 9a), which indicates that the particle convective contribution is negligible for such a small particle. Although this in accordance with the decreasing trend of α_{pc} with decrease in the particle diameter, more experimental work on different fluid-particle fluidized beds is needed in order to confirm the indication.

CONCLUSIONS

Despite of numerous available empirical correlations for heat transfer coefficient in liquid-solid fluidized beds, there is only a few studies regarding the mechanisms of heat transfer. In this work the liquid convective heat transfer mechanism in fluidized bed has been studied theoretically. A new model based on the thickness of thermal boundary layer and film theory has been proposed for the prediction of liquid convective heat transfer. The distance between two adjacent particles which collide with the wall is a key model parameter which determines the thickness of boundary layer and the fraction of heat transfer area where liquid convective mechanism takes place. The model predicts the values of liquid convective component (α_{lc}) in fluidized bed which are 4–5 times higher compared to the single-phase flow. By comparison of the model prediction with the experimental data for heat transfer coefficient for different particle sizes, it was concluded that size of the particles strongly influences the mechanism of heat transfer. For very fine particles $d_p = 0.8$ mm, the liquid convective heat transfer model represents adequately the experimental data for the overall heat transfer coefficient, indicating that the particle convective heat transfer is negligible. With the increase in particle size, the contribution of particle convective heat transfer becomes more significant, such as that for coarse particles of d_p 1.5–2 mm, in case of which the liquid convective component accounts for 60 % of the overall heat transfer coefficient.

ИЗВОД

ПРЕНОС ТОПЛОТЕ КОНВЕКТИВНИМ МЕХАНИЗМОМ У ПАРТИКУЛАТИВНО ФЛУИДИЗОВАНОМ СЛОЈУ

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У овом раду развијен је теоријски модел преноса топлоте конвективним механизмом са зида колоне на течност у партикулативно флуидизованом слоју. Модел је заснован на дебљини граничног слоја и теорији филма. Кључни параметар модела је растојање између две суседне честице које се сударају са зидом колоне. Према предложеном моделу пренос топлоте конвекцијом на течност у флуидизованом слоју је 4 до 5 пута интензивнији него у једнофазном току. Коефицијенти преноса топлоте су експериментално одређени у флуидизованом слоју вода–стаклене сфере, за различите пречнике честица. Поређење експерименталних резултата и предвиђања модела је показало да пречник честица значајно утиче на механизам преноса топлоте. За веома ситне честице пречника 0,8 mm, модел преноса топлоте конвекцијом на течност у потпуности описује

експерименталне резултате, указујући да је пренос топлоте честицама занемарљив. За крупне честице, пречника 1,5–2 mm, конвективни пренос на течност чини 60 % од укупног коефицијента преноса топлоте.

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