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SIMULATION OF VELOCITY PROFILE INSIDE TURBULENT BOUNDARY LAYER

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Abstract

The second-order differential equation with a general polynomial solution [1], is adapted for simulation of complex velocity profile inside the turbulent boundary layer. Consequently, the simulation strategy is suggested.

Keywords: velocity profile; turbulent boundary layer; second-order differential equation; polynomial solution.

Introduction

Turbulent flows are demanding for simulation due to: (i) unsteady aperiodic motion, (ii) fluid properties that exhibit 3D random spatial variations, (iii) strong dependence from initial conditions, and (iv) a wide range of scales (eddies). In other words, the turbulent simulation always has to be three-dimensional, time accurate with extremely fine grids [2-4].

Direct Numerical Simulation (DNS) under the time-dependent Navier-Stokes equations is possible only when the fluid properties reach a statistical equilibrium, for low Reynolds numbers and simple geometries. Unfortunately, the time and space details provided by DNS are not always required for design purposes.

When setting up a problem, near-wall region modeling is important because solid walls are the main source of vorticity and turbulence. Flow separation and reattachment are strongly dependent on a correct prediction of the development of turbulence near walls.

Turbulence modeling starts with following possibilities for definition of the Reynolds stresses in terms on known (averaged) quantities: (1) Boussinesq hypothesis, (2) Reynolds stress transport models, (3) non-linear eddy viscosity models (algebraic Reynolds stress), and (4) model directly the divergence of the Reynolds stresses.

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The next moment in modeling is the correct determination of the complex velocity profile inside the turbulent boundary layer. Here, a new approach and strategy will be presented.

Structure of the turbulent boundary layer

For equilibrium turbulent boundary layers, usually, we have the situation presented in Figure 1. [4]. It is well known that at high Reynolds number, the viscous dominated layer is so thin that it is challenging to resolve it.

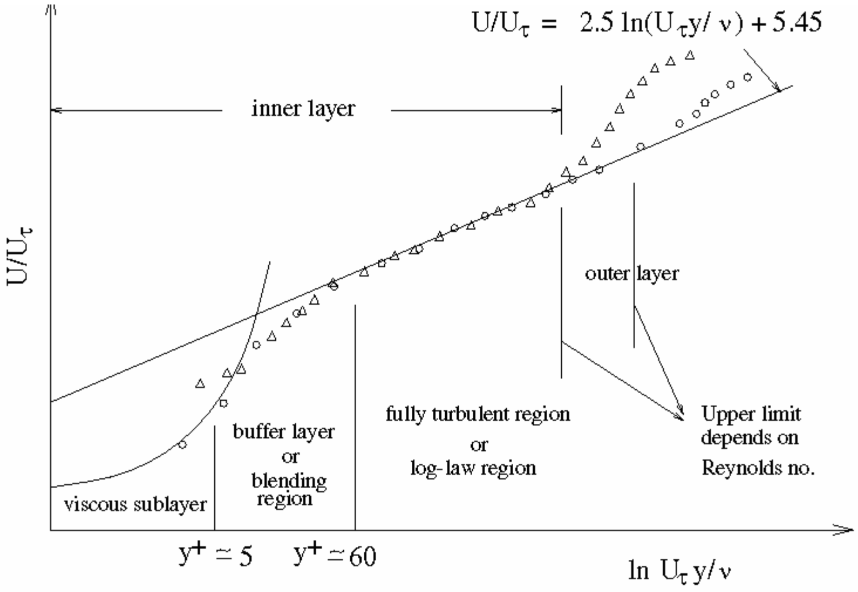

Fig. 1. Dimensionless velocity data from a wide variety of turbulent duct and boundary-layer flows [4].

In Figure 1: $y^+ = \frac{y\,U_\tau}{\nu}$, $u^+ = \frac{u}{U_\tau}$, $U_\tau = \sqrt{\frac{\tau_w}{\rho}}$

where y is the normal distance from the wall, τ_w is wall shear stress, ν is kinematic viscosity, u is velocity and ρ is density.

The mathematical formulation of a complex velocity profile

Second-order ordinary differential equation from [1], adapted for simulation of complex velocity profile inside the turbulent boundary layer is:

$$\theta''(\xi) \pm c\xi^{n} \pm f(m) = 0 \tag{1}$$

where:

$c = f_1(N)$ and $n = f_2(N)$: changeable interconnected coefficient and exponent; whole numbers or fractions.

Introducing the relevant notation for turbulent flow, the solution of equation (1) becomes the polynomial:

$$\theta = N\xi \pm \frac{1}{2} f(m)\xi^{2} \pm (N-1)\xi^{\left[\frac{N}{\pm(N-1)}\right]^{(\pm 1)*}} \tag{2}$$

where:

$\theta = \dfrac{\vartheta - \vartheta_0}{\vartheta_{\xi=1} - \vartheta_0}$: normalized dimensionless change of velocity ϑ;

$\xi = y/\delta_\chi$: dimensionless distance from surface for position χ;

δ_χ: boundary layer thickness at position (χ)

$$\delta_\chi = C_\chi \sqrt{\frac{\Pi\chi}{\vartheta_{\xi=1}}}$$

C_χ: coefficient of proportionality;

Π: kinematic and/or eddy viscosity [m^2/s];

Surface Criterion: $N = \dfrac{d\theta}{d\xi}\Big|_{\xi=0}$; is whole number or fraction, belongs [0,2];
N is determined from m (see ref. [1])

Core Criterion: $f(m) = \dfrac{d\theta}{d\xi}\Big|_{\xi=1}$; is whole number or fraction, belongs [0,±∞];
$f(m)$ is determined from m (see ref. [1])

The m is the whole characteristic number, presents the ratio of formation (F) and decomposition (D) processes inside the boundary layer, m = F/D = ± 1, 2, 3... The quantity m enables the total coupling of the analyzed situation.

In this approach, three zones exist:

1. Laminar Sublayer (LS)
2. Turbulent Fully developed zone (TF)
3. Turbulent Upper zone (TU)

Every zone has different m, N, and f(m) values, with appropriate boundaries ξ, Figure 2. and Table 1.

Table 1. An example of the equations set.

Zone	*m*	*N*	*f(m)*	*Equation*
LS [0, ξ_1]	3	1/4	-1	$\theta = \frac{3}{4}\xi - \xi^2 + \frac{1}{4}\xi^3$
TF [ξ_1, ξ_2]	7	7/8	0	$\theta = \frac{1}{8}\xi + \frac{7}{8}\xi^{\frac{1}{7}} \approx \xi^{\frac{1}{7}}$
TU [ξ_2, ξ_3]	5	5/6	9/5	$\theta = \frac{1}{6}\xi + \frac{9}{5}\xi^2 + \frac{5}{6}\xi^{\frac{1}{5}}$

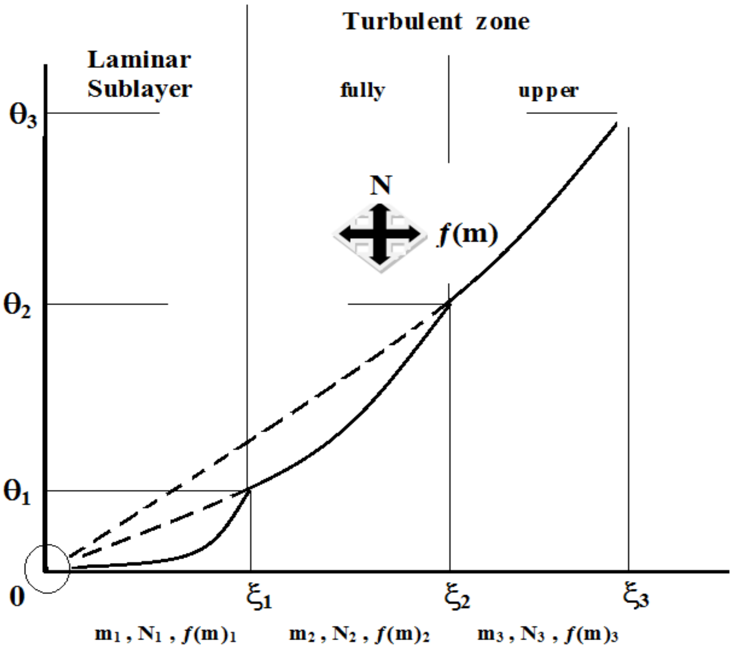

Fig. 2. Scheme of dimensionless complex velocity profile in a turbulent boundary layer.

Directions of actions of N and f(m) values on complex velocity profiles in the turbulent boundary layer are indicated in Figure 2.

Concluding remarks

Simulation of complex velocity profile in turbulent boundary layer needs a considerable computation ability, usually connected with faulty assumptions. On the other hand, experimental validation is not always a good confirmation for numerical results. Because of that, the new flexible, more simple simulation strategy is suggested.

Acknowledgments

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