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RESEARCH NOTE
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GAS – SOLID AND GAS – LIQUID MASS TRANSFER – EFFECT OF TURBULENT SCHMIDT NUMBER

The scope of this paper is to explain effect of eddy viscosity and turbulent Schmidt number on mass transfer rate. New, theoretically based correlation for gas–liquid mass transfer coefficients are proposed.
Cilj rada je bio da se objasi uticaj turbulentnog viskoziteta i turbulentnog Schmidt–ovog broja ne prenos mase. Predložene su nove korelacije za određivanje koeficijenta prealaza mase u sistemima gas–tečno. Prednost ovih korelacija je njihova teorijska zasnovanost.

Key words: Gas–liquid mass transfer, Mass transfer coefficient, Turbulent Schmidt number

The objectives of this paper is to explain the effect of turbulent Schmidt number on mass transfer coefficients.

In 1934. Gilliland and Sherwood [1] were tested Colburn's analogy (1930) [2] by measuring rates of vaporization of nine liquids into air in a wetted–wall column. These date were interpreted as a gas – solid transport, and the obtained results werer described by the following correlation:

$$Sh = 0.023\ Re^{0.83}\ Sc^{0.44} \quad (1)$$

The exponent on Schmidt number, in this correlation, was differed from 0.33 expected from Colburn analogy. Some authors [3–8] used correlation:

$$Sh = 0.023\ Re^{0.83}\ Sc^{0.33} \quad (2)$$

as a general equation for turbulent mass transfer in pipes for gas – liquid and gas – solid systems. Their explanation was that the experiments of Gilliland and Sherwood were covering small range of the Schmidt number ($0.06 \le Sc$ 2.5).

Our assumption is that different exponent on Schmidt number is not result of experimental error, it is result of different type of mass transfer. In wetted – wall columns we have gas – liquid, not gas – solid mass transfer. In wetted – wall columns mass transfer is transfer between turbulent gas core and thin liquid film, which flowing at the wall. Because of that this could not be considered as a gas – solid mass transfer. In that case, explanation of different exponents on Schmidt number could be find in eddy viscosity.

For the gas – solid systems eddy viscosity varies with a third power with the distance from the wall:

$$\epsilon_M = \gamma_3 y^3 \quad (3)$$

which leads to the exponent of 0.33 on Schmidt number.

For the fluid – fluid (gas – liquid) contact eddy viscosity varies as a second power of the distance from the wall:

$$\epsilon_M = \gamma_4\, y^2 \quad (4)$$

leading to a power 0.5 on Schmidt number. This means that gas – liquid mass transfer should be described with the correlation:

$$Sh = a\ Re^{b}\ Sc^{0.5} \quad (5)$$

Confirmation for this assumptions could be found in experimental data for wetted – wall columns (Gilliland and Sherwood [1] and Barnet and Cobe [9]). All 215 data points were correlated with equation:

$$Sh = a\ Re^{b}\ Sc^{c} \quad (6)$$

using 0.5, 0.44 and 0.33 as a exponent *c*.

The results of these correlations are shown in Table 1 as a relative errors.

Table 1. Coefficients in Equations (6) and Relative Errors

Exponent *c*	0.5	0.44	0.33
a	0.0318	0.0352	0.042
b	0.790	0.780	0.759
$\Sigma \frac{\lvert y_{cal} - y_{exp}\rvert}{y_{exp}}$	7.57 %	8.45 %	12.36 %

Clearly, relative error for the c = 0.5 is the lowest. The difference in relative errors for c = 0.5 and c = 0.44 is not significant, but for the c = 0.5 there is theoretical explanation.

This means that mass transfer in wetted wall columns is gas – liquid mass transfer, and proper correlation to describe this mass transfer should be:

$$Sh = 0.0318\ Re^{0.79}\ Sc^{0.5} \quad (7)$$

The smallest relative error was obtained for exponent on Reynolds number of 0.79, which differs

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from expected 0.83 in correlations (1) and (2). This, also, could be explained theoretically. Explanation is based on turbulent Schmidt number:

$$Sc_t = \epsilon_M / \epsilon_D \tag{8}$$

which is ratio of eddy viscosity and eddy diffusivity.

The turbulent Schmidt number is usually assumed to be constant. It was shown[10] that this assumption leads to mispresentation of the Reynolds number dependence. The turbulent Schmidt number is not constant, it changes with the changes in turbulent spectra, which shift to higher frequencies with increase in the Reynolds number. Increase of Reynolds number, not only, increase the turbulent intensity, but also moves the turbulent spectra toward higher frequencies. This means that increase in Reynolds number have a smaller effect on mass transfer than on momentum transfer rates. For the fluid flow in tubes it was found that:

$$Sc_i = a_9 Re^{0.159} \tag{9}$$

For the wetted – wall columns we assumed that the same relation will hold for the gas core in the column.

The origin of equation (7) can be found in Levich's [11] three – sublayer representation of the boundary layer, which gives the following form of correlation:

$$Sh = a_{10} Re \sqrt{f} (Sc/Sc_t)^{1/2} \tag{10}$$

If we combine equation (9) and (10) with correlated friction factor data of Gilliland and Sherwood [1]:

$$f = 0.0779 Re^{-0.254} \tag{11}$$

the resulting correlation is:

$$Sh = a_{11} R^{0.794} Sc^{0.5} \tag{12}$$

The correlation obtained from experimental data is given in equation (7). An excellent agreement between equation (7) and (12) could be seen.

From all above could be concluded that mass transfer in wetted – wall columns represent gas – liquid mass transfer, and that changes in the turbulent Schmidt number with the Reynolds number should be accounted for.

NOTATION

a	– coefficients in eqs. 5, 6, 9, 10 and 11.
b	– exponent on Reynolds number in eqs. 5. and 6.
c	– exponent on Schmidt number in eq. 6.
f	– Fanning friction factor
Re	– Reynolds number
Sc	– Schmidt number
Sc_t	– turbulent Schmidt number
Sh	– Sherwood number
y	– distance from the wall
ϵ_D	– eddy diffusivity
ϵ_M	– eddy viscosity
γ_3, γ_4	– coefficients in eqs. 3. and 4.

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