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# NUMERICAL METHOD FOR THE PREDICTION OF BENDING PROPERTIES OF GLASS-EPOXY COMPOSITES

*Marina R. Stamenović, Slaviša S. Putić, Branislav B. Bajčeta, Dragana Vitković*

*Mechanical properties of composite materials are conditioned by their structure and depend on the characteristics of structural components. In this paper is presented a numerical model by which the bending properties can be predicted on the basis of known mechanical properties of tension and pressure. Determining the relationship between these properties is justified having in mind the mechanics of fracture during bending, where the fracture takes place on the outer layer which is subjected to bending while the break ends on the layer subjected to pressure. The paper gives the values of tension, pressure and bending properties obtained by the corresponding mechanical test. A comparison of the numerical results of bending properties obtained on the basis of the model with the experimental ones, shows their satisfactory agreement. Therefore, this model can be used for some future research to predict bending properties without experiments.*

KEY WORDS: Analytical model, glass-epoxy composite material, tension test, compression test, bending properties

## INTRODUCTION

The development of modern materials is mostly based on the combination of useful properties of different materials that are combined in complex composite. These materials can satisfy the demands set by the difficult working conditions and are, therefore, considered as materials which will represent the main innovation in the future. Besides standard, and also expensive methods, analytical and numerical models are developed more and more to calculate properties without experiments. One contribution of this work is the development and testing of an analytical model by which bending properties are determined of glass-epoxy laminar composite material according to the well-known mechanical properties, tension and compression.

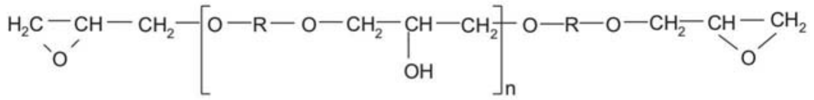
Marina Stamenović, M.Sc., Belgrade Polytechnic, Brankova 17, Belgrade, Serbia, e-mail: sasamarina@eunet.yu, Dr. Slaviša Putić, Assoc. Prof., Branislav Bajčeta, B.Sc., Dragana Vitković, B.Sc., Faculty of Technology and Metallurgy, Karnegijeva 4, 11000 Belgrade, Serbia

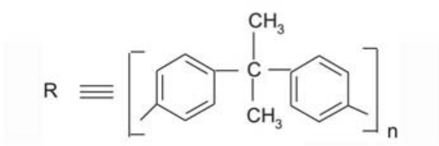
These properties can be related because the crack appears while bending on the side of the outer layer loaded on tension and the crack ends on the inner layer loaded on compression. The values of tension and compression properties used in the model for the calculation are determined by standard methods. For comparison, standard experiments on bending are also performed to check the accuracy of the model and obtained results.

## EXPERIMENTAL

### *Materials*

Basic structural components of the composite material were: glass woven (reinforcement) and epoxy resin (matrix). Glass woven on the basis of silicate glass which contains alkali up to 1% were used for reinforcement. They were also made of „E"-glass fibers which have good mechanical, hydro-thermal and electrical properties. Glass woven reinforcement of five specific masses were used: A=125 g/m², B=170 g/m², C=210 g/m², D=500 g/m² and E=880 g/m². Glass woven was made by classical procedures of spinning on different kinds of looms. Epoxy resin used as matrix material was a polycondensation product of 2,2-bis-(4-hydroxyphenyl) propane (bisphenol A) and epichlorhydrin (Epidijan 6 made by Zaklady Chemiczne „Organika-Sarzyna" S.A):





while 3-aminomethanol-3,5,5-trimethanolcyclohexylamine, a modified cycloaliphate amine of the same producer, was used for curing.

The materials were prepared by heating the resin in an oil bath to 70°C and adding the curing agent with continuous stirring until a clear homogeneous solution was obtained. Each laminate was fabricated manually in a wet lay-up. Alternate layers of fibre reinforcement plies and liquid resin were placed inside a dam on a flat mould plate. The mould plate consisting of two metal boards (upper and bottom) whose dimensions were 292 x 230 x 13 mm, was tightened up with four screws to obtain the necessary pressure force of 67 N. The material was cured during 48 h at room temperature, followed by 5 h at 90°C, and final slow cooling.

Five specific masses of glass woven reinforcement were used (A, B, C, D, E , Table 1). Two kinds of samples were cut for examination (Table 1), structure 0º/90º (samples labelled 1) and ±45º (samples labelled 2). Specimens were machined from flat panels using a high-speed diamond saw with liquid cooling. This machining operation resulted in very smooth, square cuts. One edge of each specimen was polished so that the cracks and delaminations could be readily discerned.

Mechanical tests of tension, compression and bending were performed by known standard methods (presented in Table 1) to determine the relevant mechanical properties (strength and modulus of elasticity) needed for the numerical model. Three specimens of each samples with dimensions shown in Table 1, were tested.

**Table 1.** Structures and labels of tested samples

| Test method | Sample | Number of reinforcement layers | Specific masses of reinforcement ($g/m^2$) | The layer orientation of samples | Mass fraction of reinforcement (%) | Dimensions of test specimens lxbxd (mm) |
|---|---|---|---|---|---|---|
| TENSION (ASTM D 3039) | Z-A-1 | 8 | 125 | 0°/90° | 30,0 | 250x25x2,5 |
| | Z-B-1 | 6 | 170 | | 32,8 | |
| | Z-C-1 | 5 | 210 | | 33,3 | |
| | Z-D-1 | 4 | 500 | | 55,2 | |
| | Z-E-1 | 3 | 880 | | 63,9 | |
| | Z-A-2 | 8 | 125 | ±45° | 31,6 | |
| | Z-B-2 | 6 | 170 | | 36,0 | |
| | Z-C-2 | 5 | 210 | | 37,2 | |
| | Z-D-2 | 4 | 500 | | 56,3 | |
| | Z-E-2 | 3 | 880 | | 67,2 | |
| COMPRESSION (ASTM D 3410) | P-A-1 | 24 | 125 | 0°/90° | 42,7 | 10x10x5 |
| | P-B-1 | 18 | 170 | | 33,7 | |
| | P-C-1 | 15 | 210 | | 37,1 | |
| | P-D-1 | 12 | 500 | | 57,4 | |
| | P-E-1 | 9 | 880 | | 66,3 | |
| | P-A-2 | 24 | 125 | ±45° | 41,1 | |
| | P-B-2 | 18 | 170 | | 37,1 | |
| | P-C-2 | 15 | 210 | | 38,1 | |
| | P-D-2 | 12 | 500 | | 53,7 | |
| | P-E-2 | 9 | 880 | | 68,2 | |
| BENDING (ASTM D 790) | S-A-1 | 8 | 125 | 0°/90° | 30,7 | 160x15x2,5 |
| | S-B-1 | 6 | 170 | | 33,2 | |
| | S-C-1 | 5 | 210 | | 36,3 | |
| | S-D-1 | 4 | 500 | | 55,9 | |
| | S-E-1 | 3 | 880 | | 67,3 | |
| | S-A-2 | 8 | 125 | ±45° | 31,2 | |
| | S-B-2 | 6 | 170 | | 34,7 | |
| | S-C-2 | 5 | 210 | | 37,4 | |
| | S-D-2 | 4 | 500 | | 57,1 | |
| | S-E-2 | 3 | 880 | | 67,7 | |

## RESULTS AND DISCUSSION

Tension and compression strengths of the samples were calculated from the following equation [1]:

$$R_{m(t,c)} = \frac{P_{max}}{b \cdot d} \quad [1]$$

where:

| | |
|---|---|
| $R_{m(t, c)}$ | - tension or compression strength [Pa]; |
| $P_{max}$ | - maximal tension or compression breaking force, [N]; |
| b | - width of test specimen, [m]; |
| d | - thickness of test specimen, [m]. |

Modulus of elasticity of test specimens $E_{1(t, c)}$ [GPa] was calculated by equation [2], where relations $\Delta P/\Delta\varepsilon_1$ were determined by the method of linear regression from stress-strain diagram obtained from the servo-hydraulic testing machine SCHENCK TREBEL RM 100.

$$E_{1(t,c)} = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\Delta P}{\Delta\varepsilon_1} \cdot \frac{1}{b \cdot d} \quad [2]$$

where:

$E_{1(t, c)}$ - modulus of elasticity in longitudinal direction (t-in tension; c-in compression [GPa];

$\sigma$ - stress [MPa]

$\varepsilon$ - deformation [%]

$P_{max}$ - maximal tension or compression breaking force, [N];

*b* - width of test specimen, [m];

*d* - thickness of test specimen, [m].

Examples of the force-elongation (*P-Δl*) diagrams are given in Fig. 1 (sample Z-C-1, test specimen 2) and Fig. 2 (sample Z-C-2, test specimen 1), while the force-shortening diagrams obtained by testing on compression are given in Fig. 3 (sample P-C-1, test specimen 2) and Fig. 4 (sample P-C-2, test specimen 1).

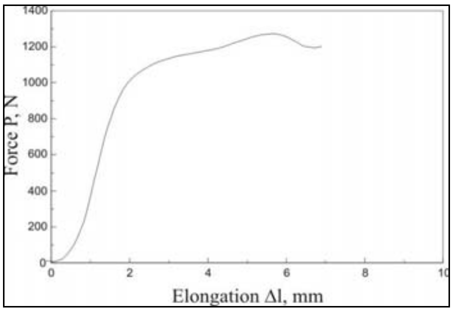



**Fig. 1.** Force-elongation diagram of tested specimen Z-C-1-2

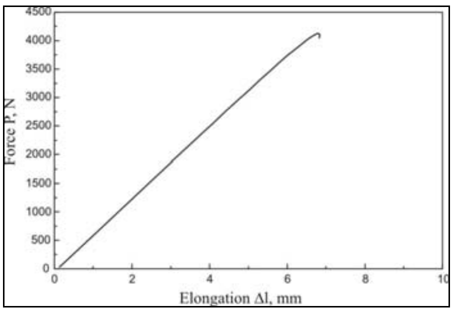



**Fig. 2.** Force-elongation diagram of tested specimen Z-C-2-1

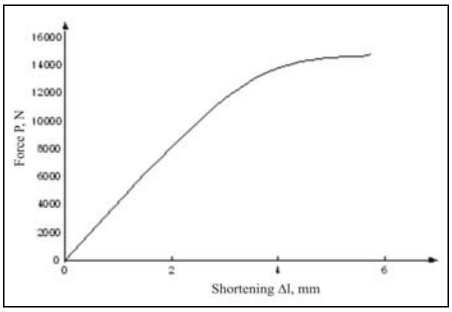



**Fig. 3.** Force-shortening diagram of tested specimen P-C-1-2

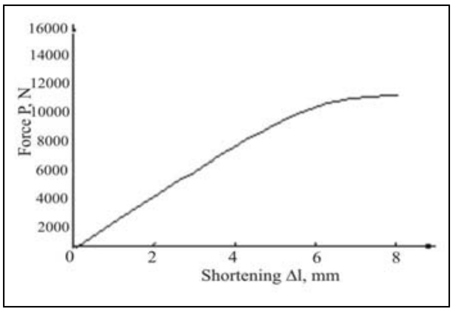



**Fig. 4.** Force-shortening diagram of tested specimen P-C-2-1

Bending strength was calculated based on the maximum bending force, $P_{max}$, from equation [3]:

$$R_f = \frac{3P_{\max} L}{2b \cdot d^2} \qquad [3]$$

where:

$R_f$ - bending strength, [MPa]

$P_{max}$ - maximum bending force, [N]

$L$ - span between the supports, [mm]

$b$ - width of test specimen, [mm]

$d$ - thickness of the test specimen, [mm]

Bending modulus of elasticity, $E_f$, was calculated from the following equation [4]:

$$E_f = \frac{L^3}{4b \cdot d^3} \cdot \frac{P}{D} \qquad [4]$$

The slope of the force-bend diagram is determined from the numerical values of $P$ and $D$, by the method of the smallest squares, where $D$ (mm) represents maximal deflection in the middle of the span of tested sample.

The force-deflection ($P$-$D$) diagram (sample S-E-1, test specimen 2) is shown in Fig. 5.

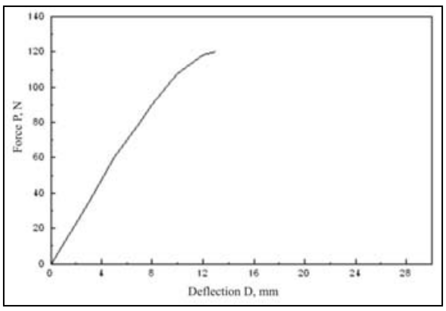



**Fig. 5.** Force-deflection diagram obtained in three-point bend test for specimen S-E-1-2

All the results are shown juxtaposed on the graphs. In Fig. 6 comparative values of tension, compression and bending strengths are given, while in Fig. 7 are given comparative moduli. In Table 2 and Fig. 8 and 9 are shown average values of the experimentally obtained strength and bending moduli needed for comparison.

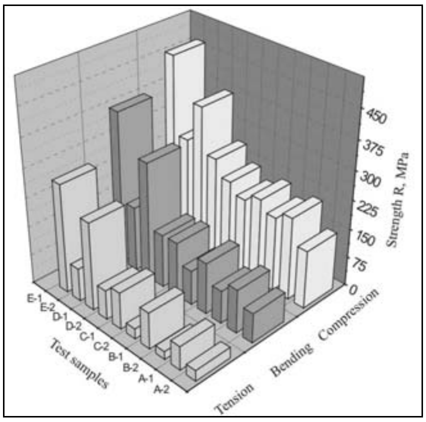



**Fig. 6.** Comparison of experimentally obtained values of tension, compression and bending strengths

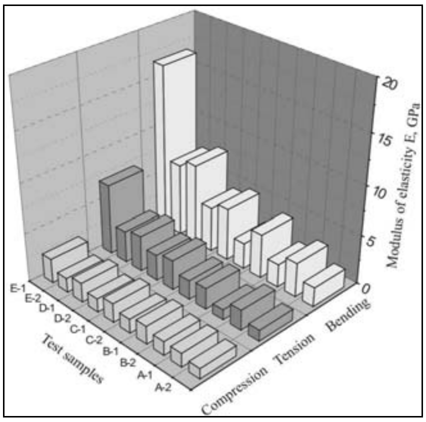



**Fig. 7.** Comparison of experimentally obtained values of tension, compression and bending modulus of elasticity

**Table 2.** Average bending values

| | SAMPLE | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0°/90° | | | | | ±45° | | | | |
| | S-A-1 | S-B-1 | S-C-1 | S-D-1 | S-E-1 | S-A-2 | S-B-2 | S-C-2 | S-D-2 | S-E-2 |
| $R_{f,av.}$ [MPa] | 112,2 | 132,6 | 139,6 | 303,5 | 393,7 | 77,2 | 84,8 | 89,4 | 137,9 | 167,5 |
| $E_{f,av.}$ [GPa] | 3,47 | 4,48 | 5,29 | 8,12 | 16,71 | 1,96 | 2,36 | 2,63 | 4,53 | 7,34 |

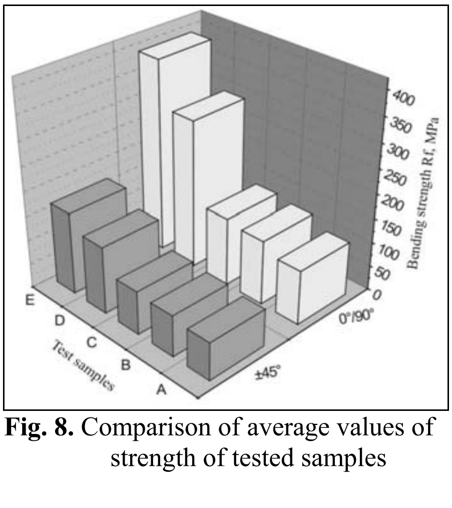



**Fig. 8.** Comparison of average values of strength of tested samples

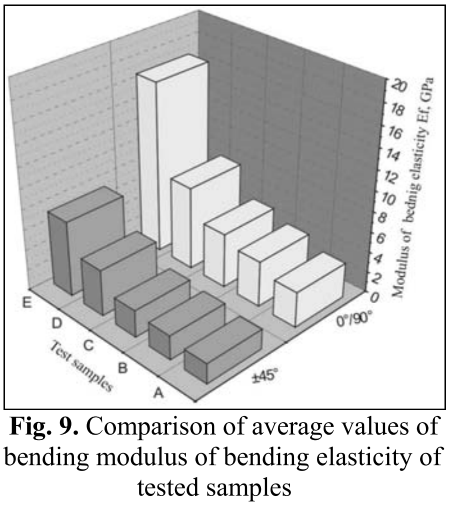



**Fig. 9.** Comparison of average values of bending modulus of bending elasticity of tested samples

*Analytical model: comparison with the test results*

As was mentioned, the aim of this work is the development of a numerical model which would predict bending properties based on tension and compression properties. The search for a relationship between these properties is fully justified having in mind the mechanics of break during bending, where the break appears on the side of the outer layer loaded on tension while the break ends on the upper (inner) layer loaded on compression. Considering that one of the most important characteristic of composite material is stiffness, that is modulus of elasticity, the first part of the analysis is based on the analytical model which allows one to determine the modulus of elasticity of the composite material on bending with the assumption of known tension and compression properties (1-5).

The basic element of analysis, which was experimentally confirmed, is that the tested composite material shows different stress-strain dependence during tests on tension and compression, and, accordingly, the axial deformation change is linear through the thickness of the test specimen, while the change is bilinear. That is why the curve of linear stress-strain dependence can be approximated by a bilinear curve, Fig. 10 (1-3, 5). The difference between the modulus of elasticity on tension ($E_{1,t}$) and compression ($E_{1,c}$) causes the movements of the neutral axis (where the values of stress and strain equal zero) which, in this case, is not in the middle of test specimen and thus influences the values of modulus elasticity of bending and bending strength (Fig. 11) (3, 5).

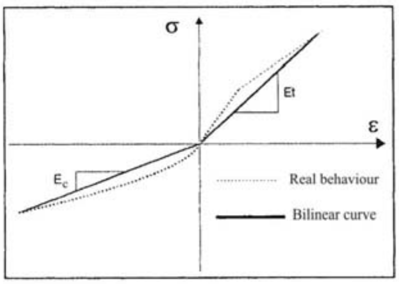



**Fig. 10.** Approximated linear stress-strain dependence with bilinear curve

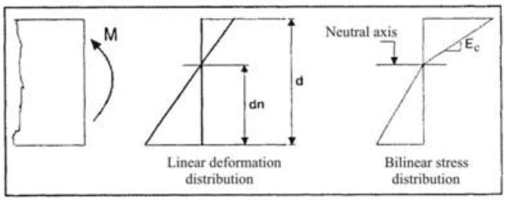



**Fig. 11.** Stress and strain distribution in the sample

Having in mind the previous analysis, as well as the relation known from equation [5] (3, 5), the bending modulus of elasticity $E_{f,calc}$ can be calculated based on the known modulus of elasticity on tension ($E_{1,t}$) and compression ($E_{1,c}$) obtained in the tests of the given composite material.

$$\frac{E_{f,calc}}{E_{1,c}} = \frac{4 \cdot \left( \frac{E_{1,t}}{E_c} \right)}{1 + 2 \cdot \left( \frac{E_{1,t}}{E_{1,c}} \right)^{\frac{1}{2}} + \frac{E_{1,t}}{E_{1,c}}} \qquad [5]$$

For a better comparison of the calculated and experimentally obtained modulus of bending elasticity, the values are shown in Fig. 12.

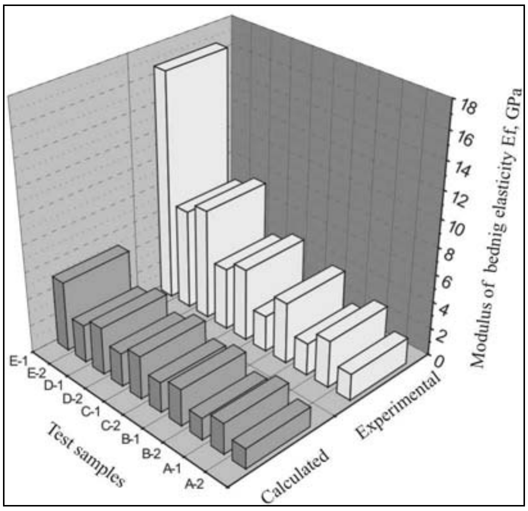



**Fig. 12.** Comparison of calculated and experimental values of the modulus of bending elasticity

In Fig. 12, certain disagreement of the values can be seen. The calculated values are lower than those obtained from the experiments. This disagreement can be explained by the approximations made during the calculation and also by the very important fact that the effects of shear stress were not taken into account, as these values were not available.

The second part of the analysis of this numeric model is based on the determination of bending strength, while knowing certain tension and compression strength and modulus of elasticity. Composite materials can be divided in four groups according to the relation of the modulus of elasticity during tension and compression and relation of tension and compression strength (3, 5):

1) $E_{1,t} > E_{1,c}$, $R_{m,t} > R_{m,c}$;
2) $E_{1,t} > E_{1,c}$, $R_{m,t} < R_{m,c}$;
3) $E_{1,t} < E_{1,c}$, $R_{m,t} > R_{m,c}$; and
4) $E_{1,t} < E_{1,c}$, $R_{m,t} < R_{m,c}$.

Observing the values obtained in the tests, it can be concluded that all samples have $E_{1,t} > E_{1,c}$ and $R_{m,t} < R_{m,c}$. In that case, the bending strength can be obtained from the following equation (3, 5):

$$R_f = \frac{R_{m,t}}{C_{t(c)}} \qquad [6]$$

where $C_t$ and $C_c$ represent corrective factors introduced because of the difference in the modulus of elasticity during tension and pressure. The values are determined based on Fig. 13 (3, 5) and with the help of the values:

$$\sigma_{t,max}=C_t \cdot \sigma_{max,ASTM} \quad ; \quad \sigma_{c,max}=C_c \cdot \sigma_{max,ASTM} \qquad [7]$$

where:

$$\sigma_{\max,ASTM}=\frac{\left\{\left[\left(\frac{E_z}{E_p}\right)^{\frac{1}{2}}+1\right]-2\right\}\cdot\frac{6\cdot P\cdot l}{b\cdot d^2}}{\frac{3\cdot P\cdot l}{2\cdot b\cdot d^2}} \qquad [8]$$

In equation [6] the smaller of calculated values of $C_t$ and $C_c$ is used.

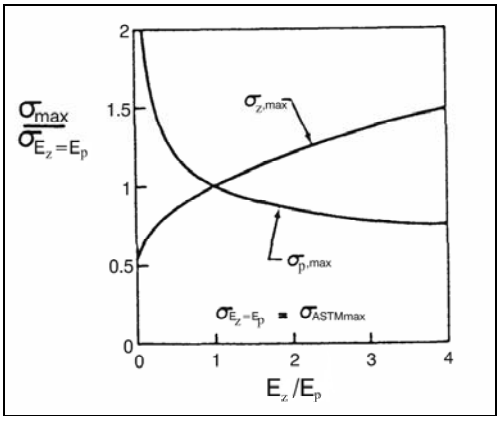



**Fig. 13.** Dependence of the corrective factors from the relation of tension and pressure modulus of elasticity

In further calculation, two corrective factors are taken into account from experience: $k_1$ and $k_2$ (1, 2, 5). They are considered with the already mentioned movements of the neutral axis in dependence of the middle level of tested sample, but the disagreements made during tests are reduced because of the big value of the bend of the tested samples (that influence the bending strength). This is how the new, corrected value of bending strength is shown in equation [9]:

$$R_{f,calc}=R_f\cdot\frac{1}{k_1}\cdot\frac{1}{k_2} \qquad [9]$$

where:

$$k_1=1+6\cdot\left(\frac{D}{L}\right)^2-4\cdot\frac{d\cdot D}{L^2} \quad ; \quad k_2=\frac{1}{2}\cdot\left[\left(\frac{E_t}{E_c}\right)^{\frac{1}{2}}+1\right] \qquad [10]$$

where:
$D$ - deflection, mm
$L$ - span between the supports, mm
$d$ - thickness of the samples, mm

Values of bending strength calculated this way and determined for all investigated structures of glass-epoxy composite material are given in Table 3 and in Fig. 14.

**Table 3.** Comparison of experimental and calculated corrected bending strength

| Sample | **Tension strength $R_{m,t}$ (MPa)** | **Pressure strength $R_{m,c}$, (MPa)** | **Experimentally determined bending strength $R_{f,\,exp}$, (MPa)** | **Calculated bending strength $R_{f,\,calc}$, (MPa)** |
|---|---|---|---|---|
| A-1 | 61.23 | 212.9 | 112.2 | 95.4 |
| A-2 | 27.50 | 150.6 | 77.2 | 62.4 |
| B-1 | 94.10 | 218.6 | 132.6 | 114.4 |
| B-2 | 20.41 | 192.5 | 84.8 | 70.3 |
| C-1 | 95.00 | 224.0 | 139.6 | 11.8 |
| C-2 | 26.68 | 196.9 | 89.4 | 71.2 |
| D-1 | 227.3 | 378.4 | 303.5 | 220.5 |
| D-2 | 75.90 | 264.1 | 137.9 | 104.1 |
| E-1 | 281.5 | 473.8 | 393.7 | 251.6 |
| E-2 | 85.45 | 277.1 | 167.3 | 118.8 |

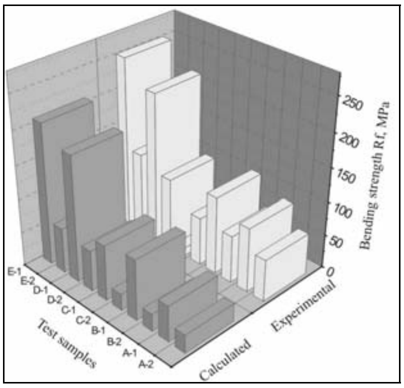



**Fig. 14.** Calculated corrected and experimentally determined values of bending strength

## CONCLUSION

On comparing results obtained analytically and experimentally it can be concluded that they show a relatively good agreement. In all cases, higher values were obtained experimentally. Considering that the disagreements are 15-30% we can not say for certain that the model is reliable. However, this model can be useful where there is a need to get approximate bending properties, and the experiments can not be performed because of the price or the lack of samples. For further analyses, these results can be just the starting information. The analysis shown here certainly should be expanded with new parameters, before all shear properties, and then the results should be confirmed by experiments, to the moment when disagreements were up to 5%.

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## ПРИМЕНА НУМЕРИЧКОГ МОДЕЛА ЗА ПРЕДВИЂАЊЕ САВОЈНИХ СВОЈСТАВА СТАКЛО-ЕПОКСИ КОМПОЗИТНОГ МАТЕРИЈАЛА


*Марина Р. Стаменовић, Славиша С. Путић, Бранислав Б. Бајчета, Драгана Витковић*



Механичка својства композитних материјала су условљена њиховом структуром и зависе од својстава структурних компонената. У овом раду приказан је нумерички модел којим се на основу познатих затезних и притисних механичких својстава могу одредити савојна својства. Довођење у везу ових својстава је оправдано имајући у виду механику лома при савијању, где до лома узорка долази са стране спољњег слоја који је оптерећен на затезање док се лом завршава на слоју оптерећеном на притисак. У раду су дате и вредности затезних, притисних и савојних својстава добијене извођењем одговарајућих експерименталних механичких испитивања. Поређењем нумеричких резултата савојних својстава добијених на основу модела са експериментално добијеним резултатима уочено је њихово задовољавајуће слагање. Самим тим показано је да се овај модел може користити у неким будућим истраживањима за приближно одређивање савојних својстава без извођења испитивања.



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