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FRICTION FACTOR FOR WATER FLOW THROUGH PACKED BEDS OF SPHERICAL AND NON-SPHERICAL PARTICLES

Article Highlights

- Experimental evaluation of pressure drop correlations in packed beds was conducted
- Pressure drop across beds of spherical and non-spherical particles was measured
- Spherical glass particles and quartz filtration sand were used as packing material
- Correlations in the form of Ergun equation gave the best results
- The coefficients in Ergun equation are system-specific

Abstract

The aim of this work was the experimental evaluation of different friction factor correlations for water flow through packed beds of spherical and non-spherical particles at ambient temperature. The experiments were performed by measuring the pressure drop across the bed. Packed beds made of monosized glass spherical particles of seven different diameters were used, as well as beds made of 16 fractions of quartz filtration sand obtained by sieving (polydisperse non-spherical particles). The range of bed voidages was 0.359–0.486, while the range of bed particle Reynolds numbers was from 0.3 to 286 for spherical particles and from 0.1 to 50 for non-spherical particles. The obtained results were compared using a number of available literature correlations. In order to improve the correlation results for spherical particles, a new simple equation was proposed in the form of Ergun's equation, with modified coefficients. The new correlation had a mean absolute deviation between experimental and calculated values of pressure drop of 9.04%. For non-spherical quartz filtration sand particles the best fit was obtained using Ergun's equation, with a mean absolute deviation of 10.36%. Surface-volume diameter (d_{SV}) necessary for correlating the data for filtration sand particles was calculated based on correlations for $d_V = f(d_m)$ and $\psi = f(d_m)$.

Keywords: pressure drop, packed bed, spherical particles, quartz filtration sand, non-spherical particles.

Packed beds of particles permit a widespread means of contact between fluid and solid phases and are used in many different industrial processes. Some examples of their application include filtration processes, ion-exchange, catalytic reactions, heat transfer, gas scrubbing, grain drying and others. The shape and size of particles that make up the bed are chosen for the characteristics of the specific process. The particle size and shape always aim at high process effectiveness, so a wide range of particles are used. In some applications, like in down-flow granular filters, polydisperse natural materials are used as the particulate phase. When natural materials are used, the shape of the particles is irregular and their size falls into some granulometric interval. Differently shaped particles pack with different degrees of bed voidage, which results in different pressure drop across the bed. The pressure drop through the packed bed is one of the most important parameters to be known for the adequate design of the process as well as for the estimation of the capital and oper-

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ating costs and sizing the pumps or fans required to force the fluid through the bed.

The pressure gradient through packed beds has been studied extensively and a large number of correlations were proposed [1-17]. The most widely used equation for pressure drop calculation was proposed by Ergun [1]:

$$-\frac{\Delta P}{H}=150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu}{d_p^2}U+1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho_f}{d_p}U^2 \quad (1)$$

The friction factor introduced by Ergun is:

$$f_p=\left(-\frac{\Delta P}{H}\right)\frac{d_p}{\rho_f U^2}\frac{\varepsilon^3}{1-\varepsilon} \quad (2)$$

According to Eqs. (1) and (2), Ergun's equation for friction factor is:

$$f_p=\frac{150}{\mathrm{Re}_\mathrm{p}^{'}}+1.75 \quad (3)$$

where:

$$\mathrm{Re}_\mathrm{p}^{'}=\frac{d_p\rho_f U}{\mu(1-\varepsilon)} \quad (4)$$

For spherical particles, d_p is the diameter of the particles that constitute the packed bed, while for non-spherical particles d_p is usually taken to be the surface-volume diameter d_{sv} [18,19]. By analogy, all of the correlations proposed for spherical particles can be used for non-spherical particles using surface-volume diameter. Note that particle sphericity is defined as:

$$\psi=\frac{d_{SV}}{d_V} \quad (5)$$

where d_V represents the volume diameter of the particle.

The other literature correlations for friction factor in packed beds of spherical and non-spherical particles are shown in Table 1. Note that some authors used different forms for friction factor and Reynolds number with respect to Ergun's definitions of f_p and $\mathrm{Re}_\mathrm{p}^{'}$, as shown in Table 1.

Table 1. Some important literature correlations for friction factor in packed beds of spherical particles

Reference	Friction factor	Eq.	Re number range
Ergun [1]	$f_p=\frac{150}{\mathrm{Re}_\mathrm{p}^{'}}+1.75$	(3)	$1<\mathrm{Re}_\mathrm{p}<2.4\cdot 10^3$
Macdonald *et al.* [2]	$f_p=\frac{180}{\mathrm{Re}_\mathrm{p}^{'}}+1.80$	(6)	-
Gibilaro *et al.* [3]	$f_p^{'}=\left(\frac{18}{\mathrm{Re}_\mathrm{p}}+0.33\right)\frac{(1-\varepsilon)}{\varepsilon^{4.8}}$; Note: $f_p=f_p{'}\varepsilon\times 3/(1-\varepsilon)$	(7)	-
Montillet *et al.* [4]	$f_\mathrm{p}^{'}=a\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left(\frac{D_c}{d_p}\right)^{0.2}\left(\frac{1000}{\mathrm{Re}_\mathrm{p}}+\frac{60}{\mathrm{Re}_\mathrm{p}^{0.5}}+12\right)$	(8)	$10<\mathrm{Re}_\mathrm{p}<2.5\cdot 10^3$ $3.8\le D_c/d_p\le 40\text{-}50$
	$a=0.061$ ($\varepsilon<0.39$), $a=0.050$ (($\varepsilon>0.39$); For (D_c/d_p) > 50, term $(D_c/d_p)^{0.2}=2.2$		
Kuerten, ref. in [5]	$f_p^{'}=\left(\frac{25\cdot 1-\varepsilon)^2}{4\varepsilon^3}\right)\left(\frac{21}{\mathrm{Re}_\mathrm{p}}+\frac{6}{\mathrm{Re}_\mathrm{p}^{0.5}}+0.28\right)$	(9)	$0.1<\mathrm{Re}_\mathrm{p}<4000$
Hicks [6]	$f_p^{'}=6.8\frac{(1-\varepsilon)^{1.2}}{\varepsilon^3}\mathrm{Re}_\mathrm{p}^{-0.2}$	(10)	$500<\mathrm{Re}_\mathrm{p}<6\times 10^4$
Tallmadge [7]	$f_p^{'}=\left(\frac{150}{\mathrm{Re}_\mathrm{p}}\frac{(1-\varepsilon)^2}{\varepsilon^3}\right)+\left(\frac{4.2(1-\varepsilon)^{1.166}}{\varepsilon^3}\mathrm{Re}_\mathrm{p}^{-1/6}\right)$	(11)	$0.1<\mathrm{Re}_\mathrm{p}<10^5$
Lee and Ogawa [10]	$f_p^{'}=\frac{1}{2}\left(\frac{12.5(1-\varepsilon)^2}{\varepsilon^3}\right)\left(\frac{29.32}{\mathrm{Re}_\mathrm{p}}+\frac{1.56}{\mathrm{Re}_\mathrm{p}^{n}}+0.1\right)$, where $n=0.352+0.1\varepsilon+0.275\varepsilon^2$	(12)	$1<\mathrm{Re}_\mathrm{p}<10^5$
Cheng [9]	$f_p=\frac{AM}{\mathrm{Re}_\mathrm{p}^{'}}+BM$, $M=1+\frac{2}{3}\frac{1}{1-\varepsilon}\frac{d_p}{D_c}$, $A=\left[185+17\frac{\varepsilon}{1-\varepsilon}\left(\frac{D_c}{D_c-d_p}\right)^2\right]\frac{1}{M^2}$ $B=\left[1.3\left(\frac{1-\varepsilon}{\varepsilon}\right)^{1/3}+0.03\left(\frac{D_c}{D_c-d_p}\right)^2\right]\frac{1}{M}$	(13a) (13)	-

Table 1. Continued

Reference	Friction factor	Eq.	Re number range
Eisfeld and Schnitzlein [10]	$f_p = \frac{154M^2}{Re_p^{'}} + \frac{M}{B}$, M by Eq. (13a), $B = \left[1.15(d_p / D_c)^2 + 0.87\right]^2$	(14)	$0.01 < Re_p < 1.76\times10^5$
Reichelt [11]	$f_p = \frac{150M^2}{Re_p^{'}} + \frac{M}{B}$, M by Eq. (13a), $B = \left[1.5\left(d_p / D_c\right)^2 + 0.88\right]^2$	(15)	-
Zhavoronkov *et al.* [12]	$f_p = \frac{165.3A^2}{Re_p^{'}} + 1.2B$, $A = B = 1 + \frac{1}{2\left(D_c / d_p\right)(1-\varepsilon)}$	(16)	-
Raichura [13]	$f_p = \frac{AM^2}{Re_p^{'}} + BM$, M by Eq. (13a), $A = \frac{103}{M^2}\left(\frac{\varepsilon}{1-\varepsilon}\right)^2\left[6(1-\varepsilon) + 80(d_p / D_c)\right]$ $B = \frac{2.8}{M}\frac{\varepsilon}{1-\varepsilon}\left[1 - 1.82(d_p / D_c)\right]^2$	(17)	-
Allen *et al.* [14]	$f_A = \left(-\frac{\Delta P}{H}\right)\frac{1}{\rho_f U^2}\left(8\frac{\sum V_p}{\sum A_p}\right)\frac{\varepsilon^3}{1-\varepsilon} = \frac{a}{Re_A} + \frac{b}{Re_A^c}$; Coefficients *a*, *b* and *c* depend on particle type and packing structure. Note: $Re_A = 4\frac{\rho_f U}{\mu(1-\varepsilon)}\frac{\sum V_p}{\sum A_p}$, $\frac{\sum V_p}{\sum A_p}\frac{d_{SV}}{6}$, $f_p = \frac{3}{4}f_A$, $Re_p = \frac{3}{2}Re_A$	(18)	$75 < Re_p < 3000$
Nemec and Levec [15]	$f_p = \frac{150}{\psi^{3/2} Re_p^{'}} + \frac{1.75}{\psi^{4/3}}$	(19)	$10 < Re_p < 500$
Singh *et al.* [16]	$f_S = \left(-\frac{\Delta P}{H}\right)\frac{d_V}{\rho_f U^2} = 4.666 Re_{pV}^{-0.2}\psi^{0.696}\varepsilon^{-2.945}\exp[11.85(\log\psi)^2]$ Note: $Re_{pV} = \frac{\rho_f U d_V}{\mu}$, $f_p = f_S \psi\varepsilon \times 3/(1-\varepsilon)$, $Re_p^{'} = Re_{pV}\psi/(1-\varepsilon)$	(20)	$1257 < Re_{pV} < 2674$
Ozahi *et al.* [17]	$f_O = \left(-\frac{\Delta P^*}{2}\right)\frac{d_V}{L}\frac{\varepsilon^3}{1-\varepsilon}\psi^2 = \frac{276}{Re_{pV}^{'}} + 1.76\psi^2$ Note: $\Delta P^* = \Delta P / \left(\frac{1}{2}\rho_f U^2\right)$, $Re_{pV}^{'} = \frac{\rho_f U d_V}{\mu(1-\varepsilon)}$, $f_p = f_O / \psi$, $Re_p^{'} = \varepsilon Re_{pV}^{'}$	(21)	$708 < Re_{pV} < 7773$

The overview of the pressure drop correlations for spherical particles shown in Table 1 is given in our previous paper [20], together with the experimental data for the friction factor for air flow through packed beds of spherical glass particles at ambient and elevated temperatures. The main conclusion of this study was that the overall best fit of all our experimental data is given by Ergun's [1] correlation, with a mean absolute deviation of 10.90% [20].

Some authors proposed friction factor correlations for non-spherical particles that included particle sphericity directly in the equations (Table 1) [14-17]. Allen *et al.* [14] reviewed the use of different correlations for pressure drop through packed beds of spherical and non-spherical particles. They have shown that the particle shape, arrangement, packing method as well as surface roughness influence the pressure drop significantly. The authors also conducted pressure drop measurements in air-particles system, using randomly packed beds of smooth and rough glass spheres, wooden cubes, wooden cylinders, acorns (ellipsoids), mixed smooth spheres of different sizes and rounded and crushed rock with equivalent diameters from 10.5 to 24.4 mm. The range of $Re_p^{'}$ numbers was 75-3000. Based on their experimental work, they proposed the correlation for non-spherical particles represented by Eq. (18), Table 1.

Nemec and Levec [15] investigated single-phase flow through packed bed reactors in the range of $Re_p^{'}$ numbers of $10 < Re_p^{'} < 500$ with dense and loose packing of different uniformly sized spherical and non-spherical particles (glass and Al_2O_3 spheres, Al_2O_3 cylinders and rings as well as Ni-Mo trilobes and quadralobes) in the size range of 1.26–3.49 mm. The fluid used in their experiments was nitrogen at 10 bar. The authors concluded that Ergun's equation represents a good approximation of the fluid flow through the packed bed of spherical particles in the investigated $Re_p^{'}$ range, while it under-predicts the pressure drop over non-spherical particles under the same conditions. For non-spherical particles they proposed a new correlation which includes the particles sphericity, given by Eq. (19), Table 1.

Singh *et al.* [16] investigated the pressure drop of air flow through packed bed solar energy storage system having large sized elements of different shapes (spheres and cubes) with d_V 125-186 mm in the range of Re_{pV} numbers from 1257 to 2674 and sphericities from 0.55 to 1. The correlation the authors proposed is given by Eq. (20), Table 1.

Ozahi *et al.* [17] investigated the pressure drop in air-particle systems, with particle size 6-19 mm, and the range of sphericity of 0.55 to 1. The range of Re_{pV} numbers in their experiments was 708-7773. The correlation for friction factor they proposed is given by Eq. (21), Table 1.

Several authors [9-13,21] investigated the influence of D_c/d_p on pressure drop in packed beds of particles with a general conclusion that the effect of D_c/d_p is negligible for D_c/d_p >10. In addition, some recent studies investigated the possibility to extend the use of Ergun's equation to polydisperse particles systems taking into account the particle size distribution [22,23].

Flow through porous media was also studied in the field of hydrology and for different applications in civil engineering. The experimental results obtained were used to validate the semi-empirical relations for non-Darcy flow [24-26].

The present study was conducted in order to investigate the optimal choice of friction factor correlation for calculating the pressure drop for water flow through packed beds of spherical and non-spherical particles. The experimental evaluation of literature correlations was conducted by measuring the pressure drop across packed beds of different packing and for different water flow rates. The spherical particles used were monosized glass beads, while the non-spherical material used was polydisperse quartz filtration sand. The values of the equivalent diameter and the sphericity of the quartz sand particles needed for the calculations were obtained by using the correlations for volume diameter and sphericity as a function of mean sieve diameter proposed in our previous paper [27]. These correlations were derived for polydisperse fractions of quartz filtration sand with sieve diameters in the d_m interval 0.359 to 2.415 mm [27]:

$$d_V = 1.0787 d_m - 0.0355 \quad (22)$$

$$\psi = 0.7942 - 0.063 d_m \quad (23)$$

where d_V and d_m are in mm.

EXPERIMENTAL APPARATUS

The experiments were performed in the water-particle system schematically shown in Figure 1. The packed bed column (f) was used for pressure drop measurements. It was equipped with a distributor and the calming section (e) in order to ensure the uniform flow of water through the bed. The upwards water flow was induced using a pump (b) and the flow rate was measured using an electromagnetic flow meter (d). The packed bed bulk temperature was measured using the temperature indicator (TI). The pressure drop in packed beds of different particles was measured using piezometers (h). The experiments were performed with two types of particles: glass spherical particles and polydisperse quartz filtration sand non-spherical particles, as shown in Figure 2. The fluid used was deaerated water at a nearly constant temperature of 20 °C. In each run, water temperature was recorded and water density and viscosity were calculated. The particle characteristics and range of the experimental conditions are summarized in Table 2.

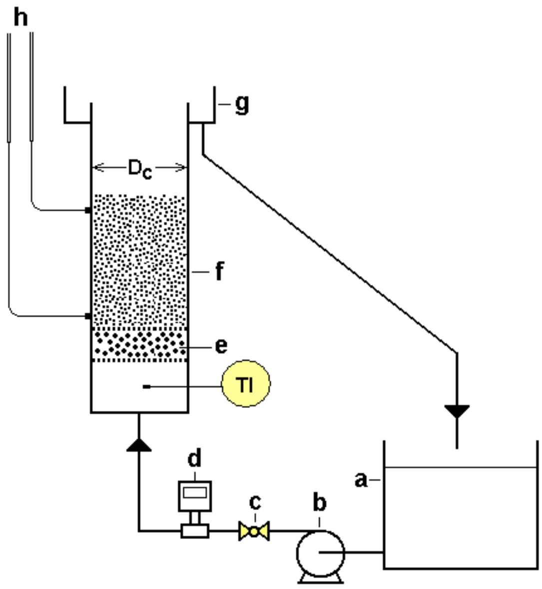

Figure 1. Schematic diagram of the experimental system (a-reservoir; b-pump; c-valve; d-electromagnetic flow meter; e-calming section; f-column; g-overflow; h-piezometers; TI-temperature indicator).

Seven kinds of mono-sized spherical glass particles were used. The experiments with glass spherical particles were conducted in two cylindrical columns: first column of the diameter of 40 mm was used for 0.840-3.020 mm particles and the second column of the diameter of 62 mm for 4.140-6.180 mm particles. The ratio of the column diameter to the particle diameter (geometric aspect ratio) in the experi-

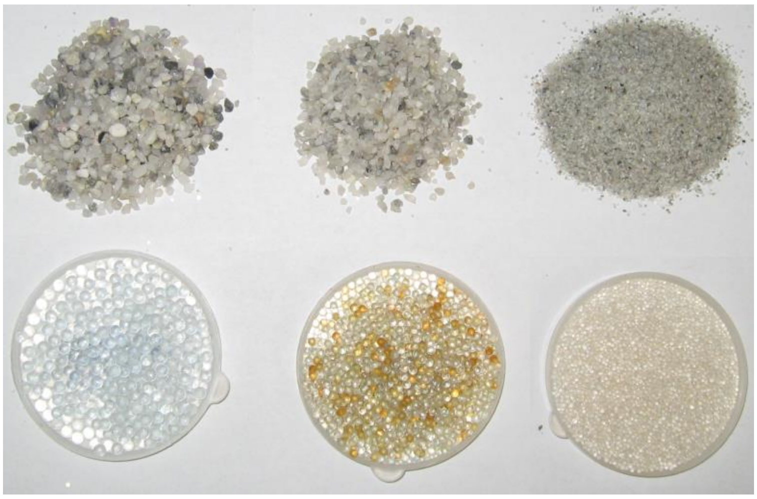

Figure 2. Some of the particles used in the experiments.

ments was between 10.0 and 47.6. As the wall effects were not studied experimentally in this paper, it should be noted that the literature on the range of geometric aspect ratio in which the wall effects are negligible is somewhat divided. Generally, the wall effects are considered negligible at D_c/d_p ratios less than 10, but there are some researchers who found wall effects to be significant at D_c/d_p ratios as high as 15-20 [10,21]. As in the case of some of our experiments the D_c/d_p ratios were in that range, the existence of wall effects cannot be excluded.

The measurements of the pressure drop in the beds of non-spherical particles were conducted using 16 fractions of quartz filtration sand obtained from the company "Kaolin"-Valjevo. The raw material was first washed by fluidization to eliminate fine dust, then dried and sieved through a number of standard sieves with sieve openings ranging from 2.830 to 0.297 mm. The obtained fractions had the sieve diameters in the interval of dm = (ds,n + ds,n+1)/ 2 = 0.359 to 2.415 mm and the ratio between the two successive sieve sizes dR =ds,n/ds,n + 1 was in the interval 1.132 to 1.715, where ds,n is the size of the opening of the sieve through which the particle had passed and ds,n+1 is the size of the opening of the sieve on which the particle was retained, as shown in Table 2.

Table 2. Particle characteristics and the range of the experimental conditions

d_p or d_m in mm	$d_{s,n+1}$ / mm	$d_{s,n}$ / mm	D_c / mm	ρ_p / kg m^{-3}	ε	U / cm s^{-1}
Spherical particles						
6.180	-	-	62	2521	0.359-0.366	0.221-2.576
5.040	-	-	62	2504	0.372-0.447	0.092-2.576
4.140	-	-	62	2514	0.370-0.442	0.097-3.163
3.020	-	-	40	2465	0.360-0.423	0.135-2.950
2.120	-	-	40	2461	0.366-0.417	0.079-2.576
1.120	-	-	40	2895	0.366-0.423	0.027-2.576
0.840	-	-	40	2875	0.367-0428	0.018-0.448
Non-spherical particles						
2.415	2.000	2.830	64	2638	0.464	0.070-1.860
1.800	1.600	2.000	64	2638	0.458	0.181-1.166
1.700	1.400	2.000	64	2638	0.453	0.098-1.036
1.583	1.166	2.000	64	2638	0.475	0.088-1.145
1.545	1.410	1.680	64	2638	0.457	0.090-1.290
1.500	1.400	1.600	64	2638	0.441	0.078-0.933
1.283	1.166	1.400	64	2638	0.449	0.067-0.943
1.201	0.991	1.410	64	2638	0.500	0.046-1.178
1.098	1.030	1.166	64	2638	0.461	0.052-0.984
1.000	0.750	1.250	64	2638	0.426	0.037-0.423

Table 2. Continued

d_p or d_m in mm	$d_{s,n+1}$ / mm	$d_{s,n}$ / mm	D_c / mm	ρ_p / kg m^{-3}	ε	U / cm s^{-1}
Non-spherical particles						
0.940	0.850	1.030	64	2638	0.468	0.067-0.788
0.781	0.711	0.850	64	2638	0.470	0.015-0.626
0.656	0.600	0.711	64	2638	0.464	0.017-0.254
0.560	0.519	0.600	64	2638	0.471	0.033-0.298
0.505	0.420	0.589	64	2638	0.483	0.011-0.321
0.359	0.297	0.420	64	2638	0.486	0.011-0.091

A total of 23 runs were conducted (7 with spherical particles and 16 with non-spherical particles) and a total of 725 data points were collected (511 for spherical and 214 for non-spherical particles). The bed particle Reynolds number, $Re_p^{'}$, varied between 0.3 and 286 for spherical particles and between 0.1 and 50 for non-spherical particles. All of the water superficial velocities used in the experiments were below the minimum fluidization velocity for the respective particles. The velocities were in the range of 0.03-0.94 U_{mF}, where U_{mF} represents minimum fluidization velocity.

RESULTS AND DISCUSSION

The results of the friction factor f_p *vs.* $Re_p^{'}$ obtained by the experimental measurements of pressure drop are shown in Figures 3 and 4 for spherical and non-spherical particles, respectively. The comparison between the experimental results and the selected literature correlations is given in Table 3 and in Figures 5 and 6. The literature correlations for spherical particles shown in Table 1 were tested for all the experimental data. The surface-volume diameter and the sphericity needed for the calculations for non-spherical particles were obtained using the correlations from our previous paper [27] for quartz filtration sand (Eqs. (22) and (23)). For quartz filtration sand beds, the literature correlations specifically defined for non-spherical particles were also tested: Allen *et al.* [14], Nemec and Levec [15], Singh *et al.* [16] and Ozahi *et al.* [17] correlations.

The mean absolute deviation between the measured values of the pressure gradient and the values obtained from the literature correlations were calculated according to the following equation:

$$\sigma = \frac{1}{N}\sum_{1}^{N}\left|\frac{(\Delta P / H)_{\text{calc}} - (\Delta P / H)_{\text{measured}}}{(\Delta P / H)_{\text{measured}}}\right| \quad (24)$$

where N is the number of data points, $(\Delta P/H)_{\text{calc}}$ and $(\Delta P/H)_{\text{measured}}$ are the calculated and the measured pressure gradients.

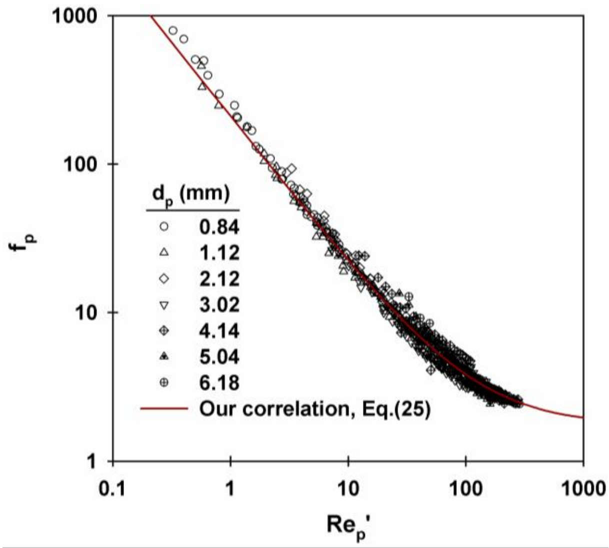

Figure 3. f_p vs. $Re_p^{'}$ for spherical particles.

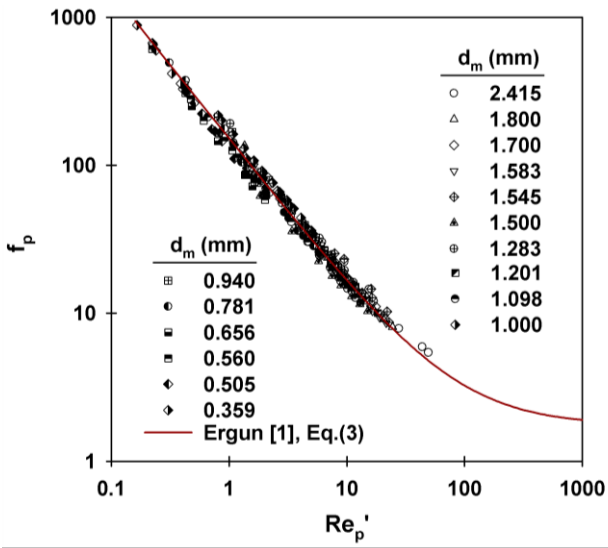

Figure 4. f_p vs. $Re_p^{'}$ for non-spherical particles.

As can be seen from Table 3, the best fit of our experimental data for pressure drop in beds of spherical particles was obtained using Cheng [9] correlation, with mean absolute deviation of 10.89%. The correlations of Macdonald *et al.* [2] and Montillet *et al.*

[4] also gave very good results in fitting our experimental data with mean errors of 12.18 and 13.13%, respectively. A number of other correlations tested gave results with mean absolute deviation in the range of 16-20%. It should be noted that Hicks [6] correlation gave the mean error of 50.19% for spherical particles and 84.34% for non-spherical particles. The reason for such a large error is that the range of Re_p numbers in this paper was below 183, while the specified range of applicability of Hicks correlation is $Re_p > 500$.

Table 3. Comparison of experimental data for friction factor, σ (%), with different correlations from the literature

Reference	Particles	
	Spherical	Non-spherical
Ergun [1]	19.82	10.36[a]
Macdonald *et al.* [2]	12.18	21.28[a]
Gibilaro *et al.* [3]	18.19	13.28[a]
Montillet *et al.* [4]	13.13	45.53[a]
Kuerten, ref. in [5]	29.36	24.85[a]
Hicks [6]	50.19	84.34[a]
Tallmadge [7]	16.04	11.19[a]
Lee and Ogawa [8]	18.61	25.30[a]
Cheng [9]	10.89	31.74[a]
Eisfeld and Schnitzlein [10]	18.78	11.31[a]
Raichura [13]	39.07	111.39[a]
Reichelt [11]	20.02	10.64[a]
Zhavoronkov *et al.* [12]	19.04	13.58[a]
Allen *et al.* [14]	-	59.40[b]
Nemec and Levec [15]	-	63.52[c]
Singh *et al.* [16]	-	85.22
Ozahi *et al.* [17]	-	78.53
This paper (Eq.(25))	9.04	-

[a]Using d_{SV}; [b]Using coefficients for crushed rock, d_{SVn} = 24.4 mm; [c]Using coefficients for cylindrical particles

The correlations with mean absolute deviation between experimental and correlated pressure drop less than 20% for beds of spherical particles are shown in Figure 5. The correlations shown in Figure 5 were calculated for D_c/d_p = 25 and bed porosity of ε = 0.40 (the mean values in our experiments) in order to be able to show the correlations with direct dependence on ε as lines.

Compared to the data of our previous paper [20], in which air-spherical particles system was investigated, the results are in the similar range for Macdonald *et al.* [2] (~12%) and Cheng [9] (~11 and 12%) correlations, which gave very good results at ambient temperature both for air-particles and water-particles systems. On the other hand, the correlations of Ergun [1], Tallmadge [7], Reichelt [11], Gibilaro *et al.* [3], Einsfeld and Schnitzlein [10] and Zhavoronkov *et al.* [12] performed better in air-particles system, while the correlation of Montillet *et al.* [4] performed better in water-particles systems.

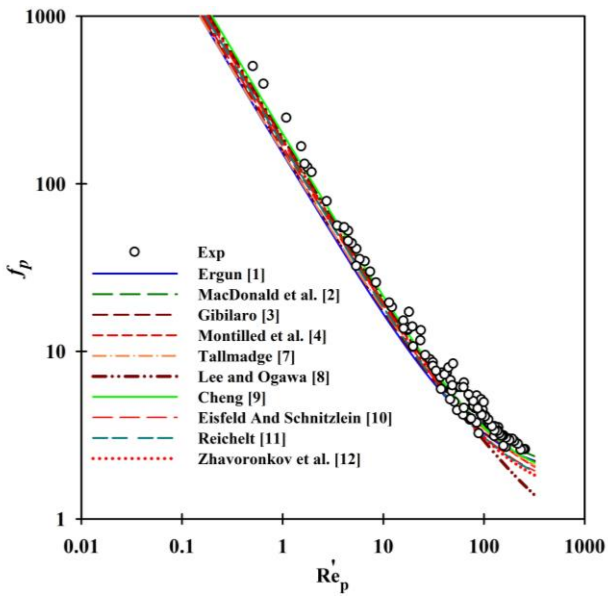

Figure 5. Comparison of experimental data of f_p vs. Re'_p with chosen correlations for spherical particles.

In order to improve the correlation results for spherical particles, a new simple equation is proposed in the form of Ergun's equation, with modified coefficient:

$$f_p = \frac{209}{Re'_p} + 1.75 \quad (25)$$

The mean absolute deviation between the values calculated from Eq. (25) and the experimental data is 9.04%. The new correlation is shown in comparison to the experimental data in Figure 3.

For non-spherical particles (quartz filtration sand), the best fit of the experimental data was obtained using Ergun's equation [1], with mean absolute deviation of 10.36%. The correlations of Reichelt [11], Tallmadge [7] and Einsfeld and Schnitzlein [10] also gave very good results in fitting the experimental data with mean errors of 10.64, 11.19 and 11.31%, respectively. The correlations with mean absolute deviation of pressure drop less than 20% for quartz filtration sand packed beds are shown in Figure 6. The correlations shown in Figure 6 were calculated for D_c/d_p = 73 and bed porosity of ε = 0.40 (the mean values in our experiments) in order to be able to show the correlations with direct dependence on ε as lines.

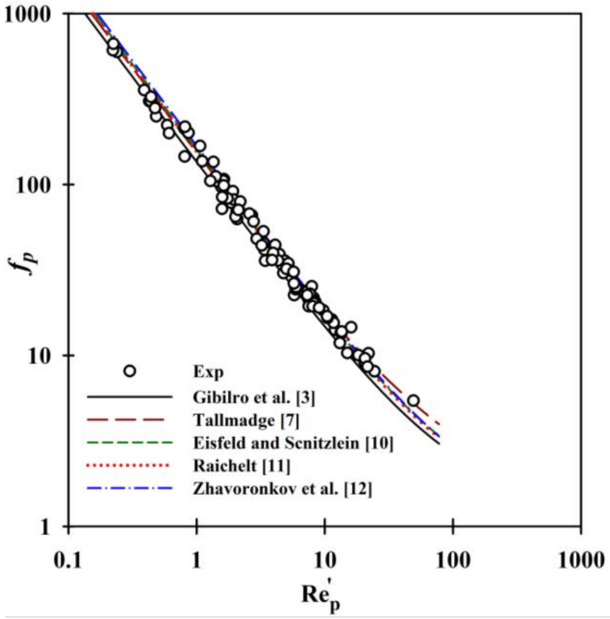

Figure 6. Comparison of experimental data of f_p vs. Re'_p with chosen correlations for non-spherical particles.

The correlations specifically derived for non-spherical particles gave very poor results in correlating our experimental data. The mean absolute deviations for these correlations were in the range of 59.40 to 85.22%. The reason for this is that the correlations of Allen *et al.* [14], Sing *et al.* [16] and Ozahi *et al.* [17] were derived from experiments in systems in which the particles were of large diameters (6-20 mm equivalent diameters) and the Re_p numbers were larger than 75, while in our system, the Re_p numbers of non-spherical particles were smaller than 25. Allen *et al.* [14] and Singh *et al.* [16] correlations were derived for packed bed solar energy storage systems with particle materials being wooden cubes, crushed rock, concrete and masonry bricks. On the other hand, the correlation of Nemec and Levec [15] was derived for the particles in the range of 1.26-3.49 mm, but in the high pressure system (5-20 bar).

The very poor performance of the correlations derived for the non-spherical particles in our quartz sand packed beds emphasizes the fact that the friction factor in packed beds strongly depends on other variables besides the sphericity of the particles. It can be concluded that the friction factor is very system specific and that it depends on particle shape and size, voidage, arrangement, packing method as well as surface roughness.

It is interesting to note that almost all of the correlations that gave good results in correlating our experimental data both for spherical and non-spherical particles were in the form of Ergun's equation, *i.e.*, represented a modification of this equation with different coefficients. From this fact it can be concluded that the form of Ergun's equation with two added terms describing viscous and inertial effects is adequate for representing the friction factor in packed beds. However, the coefficients in the equation are very system-specific and care should be taken when choosing the adequate equation for pressure drop calculation for the specific purpose. The correlation chosen for the specific system should be obtained from the data in the similar range of experimental conditions as the system it is intended to be applied to.

CONCLUSIONS

The present study was conducted in order to investigate the optimal choice of friction factor correlation for water flow through packed bed of particles. The best fit of experimental data for spherical particles was obtained using the Cheng [9] correlation (mean absolute deviation of 10.89%). Ergun's equation [1] gave better results in correlating the data for non-spherical particles with mean absolute deviation of 10.36% compared to 19.82% for spherical glass particles. Ergun's equation was modified in order to improve the fit for spherical particles and a new correlation was proposed. The mean absolute deviation between the experimental data and the proposed correlation is 9.04%.

The correlations specially derived for non-spherical particles gave very poor results in correlating our experimental data probably because they were derived for systems with much larger particles packed in a different arrangement. Most of the correlations that gave good results in correlating experimental data both for spherical and for non-spherical particles were in the form of Ergun's equation with modified coefficients, thus showing that the form of Ergun's equation with two added terms describing viscous and inertial effects is adequate for representing the friction factor in packed beds. However, the coefficients in the equation are very system-specific.

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Nomenclature

a	empirical coefficient in Eq. (6), dimensionless
A_p	area of a particle
b	empirical coefficient in Eq. (6), dimensionless
c	empirical coefficient in Eq. (6), dimensionless
d_m	particle sieve diameter
d_p	packed bed particle diameter

d_s sieve opening
d_{SV} particle surface-volume diameter
$d_{s,n}$ the size of the opening of the sieve through which the particle had passed
$d_{s,n+1}$ the size of the opening of the sieve on which the particle was retained
d_V particle volume diameter
D_c column diameter
f_A friction factor defined according to Eq. (18)
f_o friction factor defined according to Eq. (21)
f_S friction factor defined according to Eq. (20)
f_p friction factor defined according to Eq. (2)
f'_p modified friction factor defined according to Eq. (7)
H bed height
P pressure
ΔP bed pressure drop, *Pa*
ΔP^* dimensionless bed pressure drop, Eq.(21)
Re_p $=(\rho_f U d_p)/\mu$ particle Reynolds number
Re'_p $=(\rho_f U d_p)/(\mu(1-\varepsilon))$ bed particle Reynolds number
Re_{pV} particle Reynolds number defined according to Eq. (20)
Re_{pV}' bed particle Reynolds number defined according to Eq. (21)
Re_A bed particle Reynolds number defined according to Eq. (18)
U superficial fluid velocity
U_{mF} minimum fluidization velocity (superficial)
V_p volume of a particle

Greek letters

ε voidage
μ fluid viscosity
ρ_f fluid density
ρ_p particle density
σ mean absolute deviation
ψ particle sphericity, dimensionless

Subscripts

f fluid
mF minimum fluidization
p particle

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NAUČNI RAD

KOEFICIJENT TRENJA FLUID-ČESTICE U PAKOVANIM SLOJEVIMA SFERIČNIH I NESFERIČNIH ČESTICA

Cilj ovog rada je bio eksperimentalno ispitivanje koeficijenta trenja fluid-čestice prilikom strujanja vode kroz pakovani sloj sferičnih i nesferičnih čestica na sobnoj temperaturi. U eksperimentima je meren pad pritiska prilikom strujanja fluida kroz pakovani sloj. Na osnovu dobijenih rezultata, izvršena je evaluacia različitih literaturnih korelacija. U eksperimentima je korišćeno sedam vrsta monodisperznih sferičnih staklenih čestica, kao i 16 frakcija polidisperznih nesferičnih čestica filtracionog peska različitih dimenzija, dobijenih prosejavanjem. Opseg poroznosti pakovanih slojeva je bio od 0,359 do 0,486, dok je opseg vrednosti Rejnlodsovog broja za čestice bio od 0,3 do 286 za sferične čestice i od 0,1 do 50 za nesferične čestice. Dobijeni rezultati su korelisani korišćenjem većeg broja literaturnih korelacija. U cilju poboljšanja rezultata korelisanja za sferične čestice, predložena je nova jednačina u formi Ergunove jednačine sa modifikovanim koeficijentima. Srednje apsolutno odstupanje eksperimentalnih od izračunatih vrednosti za predloženu korelaciju iznosilo je 9,04%. Za nesferične čestice kvarcnog filtracionog peska, najbolji rezultati su dobijenu korišćenjem Ergunove jednačine, sa srednjim apsolutnim odstupanjem od 10,36%. Površinsko-zapreminski prečnik (d_{SV}) koji je neophodan za korelisanje eksperimentalnih podataka za nesferične čestice je računat na osnovu korelacija za $d_V = f(d_m)$ i $\psi = f(d_m)$ koje su predložene u našem prethodnom radu [27].

Ključne reči: pad pritiska, pakovani sloj, sferične čestice, kvarcni filtracioni pesak, nesferične čestice.