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# INFLUENCE OF ENCASTERING ON THIN-WALLED CANTILEVER BEAMS WITH U AND Z PROFILES ON THE MAGNITUDE OF EQUIVALENT STRESS AND DEFORMATION

# UTICAJ OBLIKA UKLEŠTENJA TANKOZIDIH KONZOLA OBLIKA U I Z PROFILA NA VELIČINU EKVIVALENTNOG NAPONA I DEFORMACIJE

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**Keywords**
- thin-walled beams
- equivalent stress
- finite element
- deformation

*Abstract*

*The purpose of this work is to present analytical and numerical determination of equivalent stress and deformation of open section thin-walled U and Z cantilever beams loaded with torsion. This work can be divided into two parts. In the first part of this paper, equivalent stress and deformation are obtained by analytical calculation for encastered model over the whole cross section. In the second part, the finite element method is applied for four different encastered models and the obtained results are compared with the analytical calculation.*

**Ključne reči**
- tankozidi nosači
- ekvivalentni napon
- konačni element
- deformacija

*Izvod*

*Cilj ovog rada jeste da prikaže analitičko i numeričko određivanje ekvivalentnog napona i deformacije kod U i Z tankozidih konzola otvorenog poprečnog preseka, opterećenih na uvijanje. U radu se mogu izdvojiti dve celine. U prvom delu su analitičkim putem dobijeni ekvivalentni napon i deformacija za model sa ukleštenjem po celom preseku, a u drugom delu je primenjena metoda konačnih elemenata za četiri različita modela ukleštenja i dobijeni rezultati su upoređeni sa analitičkim proračunom.*

## INTRODUCTION

Thin-walled beams find a wide application in construction and machinery industry, as they enable obtaining any shape of beam cross-section. Due to their low weight, thin-walled open section beams are widely applied in many structures. Many modern metal structures are manufactured using thin-walled elements (shells, plates, thin-walled beams) subjected to complex loads, /1/. In most structures such as automotive, railway, vehicles, boats and similar, they are installed in thin-walled elements. Thin-walled elements can be various shapes, can have greater or lesser bending and torsional rigidity, but their common property is that they have low weight compared to other possible structural shapes, /2-4/.

## ANALYTICAL CALCULATION

Properties of the material used in this paper are given in Table 1, /3/.

Table 1. Mechanical properties of steel S235.

| Modulus of elasticity (Pa) | Poisson's ratio | Yield stress (Pa) | Allowable stress (Pa) |
|---|---|---|---|
| $2.1 \cdot 10^{11}$ | 0.3 | $235 \cdot 10^{6}$ | $160 \cdot 10^{6}$ |

Figure 1 shows cross-sections of thin-walled elements, where: $b_1 = b_3 = 81.5$ mm are bandwidths; $b_2 = 103$ mm is rib height; and $t = 3$ mm is thickness.

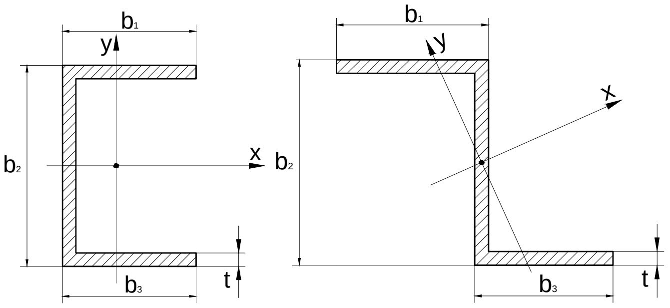

Figure 1. Cross-sections U and Z of cantilever beam.

Area size of the cross-section is calculated using expression /5, 6/:

$$A = \sum_{i=1}^{3} b_i t_i \; . \qquad (1)$$

Moments of inertia of the cross-sectional area about the centroidal axes $x$ and $y$ are given by expressions, /6/:

$$I_x = \sum_{i=1}^{3} t_i \int y(s)y(s)ds \ , \tag{2}$$

$$I_y = \sum_{i=1}^{3} t_i \int x(s)x(s)ds \ . \tag{3}$$

Sectorial moment of inertia is given by expression /6/:

$$I_\omega = \int_A \omega^2 dA = \sum_{i=1}^{3} t_i \int_S \omega(s)\omega(s)dS \ . \tag{4}$$

Torsional moment of inertia is given by expression /6/:

$$I_t = \frac{\eta}{3} \sum_{i=1}^{3} b_i t_i^3 \ , \tag{5}$$

where: $\eta$ is coefficient of safety.

Torsional section module is given by expression:

$$W_t = \frac{I_t}{t_{\max}} \ . \tag{6}$$

Schematic representation of constrained torsion of the cantilever beam is given in Fig. 2.

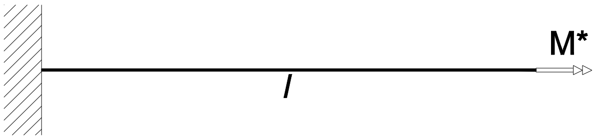

Figure 2. Constrained torsion of the cantilever beam.

The cantilever beam is loaded with a torsion moment according to the expression:

$$M^* = 15700 \text{ Nmm} \ . \tag{7}$$

The reduced Young's modulus is given by expression:

$$\bar{E} = \frac{E}{1-\nu} \ . \tag{8}$$

The bending-torsional characteristic is given by /6/:

$$k = \sqrt{\frac{GI_t}{\bar{E}I_\omega}} \ . \tag{9}$$

Bimoment and maximum normal stress are given by Eqs.(10) and (11), respectively, /1, 2/

$$B_{\max} = -\frac{M^*}{k} \text{th}(kl) \ , \tag{10}$$

$$\sigma_{\max} = \frac{B_{\max}}{I_\omega} \omega_{\max} \ . \tag{11}$$

In the case of loads by concentrated torsion moment on the free end of the cantilever beam, the moment of pure torsion on the free end is given by expression, /3/:

$$M_{t\max} = M^* \left( 1 - \frac{1}{\text{ch}(kl)} \right) . \tag{12}$$

The shear stress is given by expression:

$$\tau_{\max} = \frac{M_{t\max}}{W_t} \ . \tag{13}$$

In the case of a complex load (normal and shear stress are taken together in the calculation), the equivalent stress, calculated by Hencky-Mises hypothesis /6/, can be defined:

$$\sigma_e = \sqrt{\sigma_{\max}^2 + 3\tau_{\max}^2} \ . \tag{14}$$

Based on the previous equations and equations presented in literature /1, 2/, the geometrical characteristics of cross sections of the given cantilever beam (Fig. 1) are obtained and are presented in Table 2.

Table 2. Geometrical characteristics of cross sections.

| Profile | U | Z |
|---|---|---|
| $A$ (cm$^2$) | 7.8 | 7.8 |
| $I_x$ (cm$^4$) | 145.1 | 145.1 |
| $I_y$ (cm$^4$) | 55.2 | 102.42 |
| $W_x$ (cm$^3$) | 28.17 | 28.17 |
| $W_y$ (cm$^3$) | 9.97 | 12.80 |
| $I_t$ (cm$^4$) | 0.262 | 0.262 |
| $W_t$ (cm$^3$) | 0.873 | 0.873 |
| $I_\omega$ (cm$^6$) | 971.03 | 1377.3 |

According to Eqs.(1-14) and equations given in literature /7, 8/, the normal, tangential, equivalent stresses and deformations are obtained. The obtained values are given in Table 3. The models are designed to have the same cross-sectional area size and are loaded with the same intensity of the torsion moment. The lengths of the cantilever beams are $l$ = 1000 mm.

Table 3. Stress and strain.

| Profile | U | Z |
|---|---|---|
| $M^*$ (Nmm) | 15700 | 15700 |
| $B_{\max}$ (Nmm$^2$) | 12114000 | 13061000 |
| $\sigma_{\max}$ (MPa) | 29.25 | 36.46 |
| $\tau_{\max}$ (MPa) | 2.16 | 4.67 |
| $\sigma_e$ (MPa) | 29.5 | 37.3 |
| $\theta_{\max}$ (°) | 0.97 | 0.712 |

## NUMERICAL ANALYSIS USING FINITE ELEMENT METHOD

Numerical simulations /9-14/ have been performed using KOMIPS software. The used units are (mm) and (N). In Figs. 3 and 4, load and boundary conditions /9/ are shown. The shell elements are used (shell model), /9/. The torsion moment is introduced through the coupling of the forces $F$ = 157 N through the centre of gravity of the cross-section, and the moment they create is $M^*$ = 15.7 Nm.

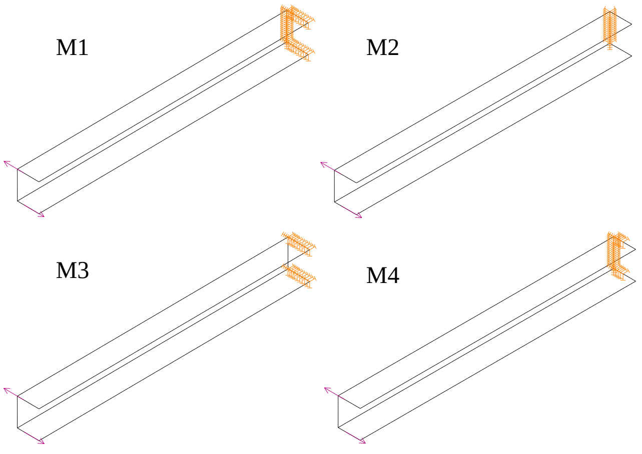

Figure 3. Load and boundary conditions in the U profile.

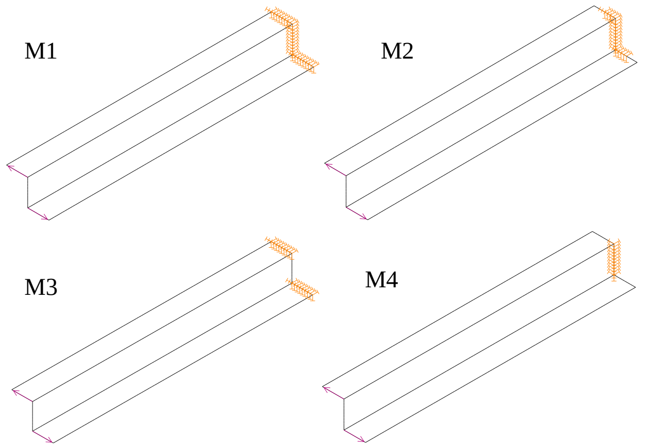

Figure 4. Load and boundary conditions in the Z profile.

## RESULTS AND DISCUSSION

The model displacements are shown in Figs. 5 and 6, and the maximal displacement $f_{max}$ is given for each model in millimetres. Figure 7 shows the distribution of equivalent Hencky-Mises stress for the U profile.

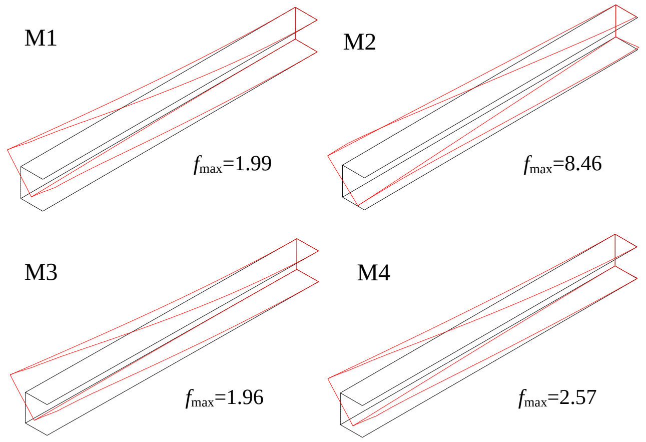

Figure 5. Deformed U profile model with maximal displacement (mm).

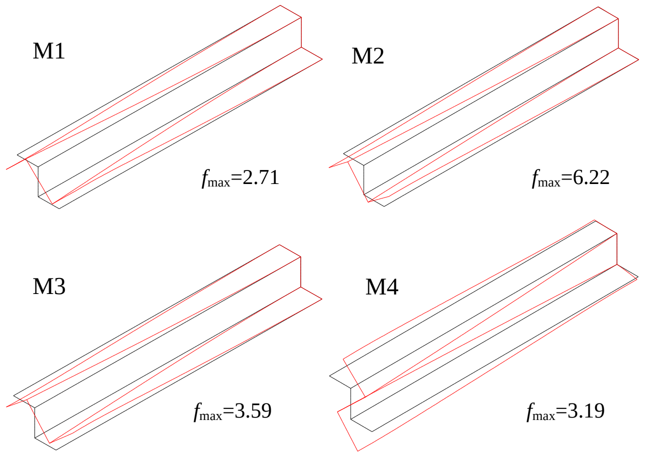

Figure 6. Deformed Z profile model with maximal displacement (mm).

For the cross-sections of U profile thin-walled cantilever beam, four models (M1 to M4) are considered (Fig. 3). The stresses obtained analytically, shown in Table 3 for the U profile, and obtained numerically (Fig. 7) by applying the finite element method, differ by about 8 %. The M2 model (Fig. 5) has the most disadvantageous way of reliance. Models M1 and M3 have the most convenient encastering method.

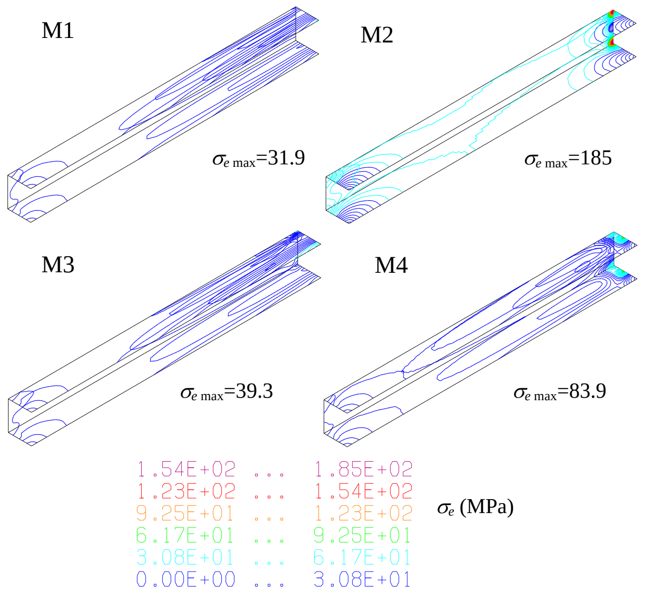

Figure 7. Equivalent Hencky-Mises stress, U profile.

For thin-walled cantilever beams loaded with a torsion moment at the free end, the deformation and the Von Mises stress depends on the encastering model. The thin-walled cantilever beams with cross sections of U and Z profiles and the various ways of encastering are discussed. For both profiles, four models (M1, M2, M3 and M4) are considered. All models have different types of encastering, i.e. different length and position of encastering.

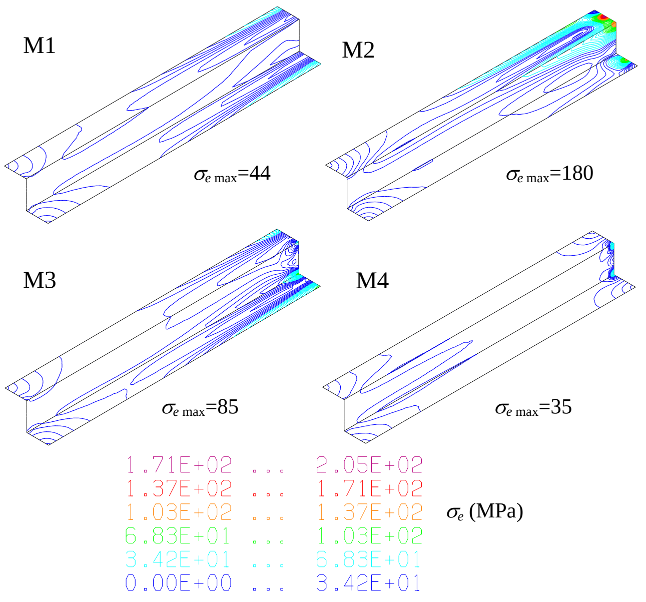

Figure 8. Equivalent Hencky-Mises stress, Z profile.

Model M1 of Z profile is encastered over the whole cross section and has the lowest value of maximal deformation. Values of the equivalent stress obtained analytically, shown in Table 3 for the Z profile, and obtained numerically, using finite element method (Fig. 8), differ by 15 %.

The M2 model (Fig. 8) has the most disadvantageous form of encastering. Model M3 has a slightly higher stress value than model M1. Model M4 (Fig. 8) has the most favourable way of encastering that provides a stress reduction of about 20 % over model M1.

## CONCLUSIONS

The paper provides initial considerations that include an overview of current studies of the stress and deformation states of thin-walled beams, as well as a review of available literature. In this paper, computational models are made, the static calculation is carried out analytically and using the finite element method. The zones of stress concentration are identified and the process of stress reduction and its concentration is presented. The conclusions obtained by examining this type of structure may be involved in the design process of new similar structures. Findings obtained during the implementation of this work can be directly applied to identify the behaviour of real structures in their working conditions, i.e. in exploitation.

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