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AUTHORS' REVIEW

**A mathematical model of the current density distribution
in electrochemical cells**

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Abstract: An approach based on the equations of electrochemical kinetics for the estimation of the current density distribution in electrochemical cells is presented. This approach was employed for a theoretical explanation of the phenomena of the edge and corner effects. The effects of the geometry of the system, the kinetic parameters of the cathode reactions and the resistivity of the solution are also discussed. A procedure for a complete analysis of the current distribution in electrochemical cells is presented.

Keywords: current distribution; edge effect; corner effect.

CONTENTS

1. BASIC FACTS
2. A STATEMENT OF THE PROBLEM
3. THE CURRENT DENSITY DISTRIBUTION IN HOMOGENOUS FIELDS
4. THE EDGE EFFECT
5. TWO EQUAL PLANE PARALLEL ELECTRODES ARRANGEMENT
 - 5.1. Ohmic resistance of the cell
 - 5.2. The very edge ohmic resistance
 - 5.3. The edge effect on the current density distribution
 - 5.4. The depth of the penetration of a current line between the electrode edges and the cell side walls
 - 5.5. The dendritic growth initiation at the edges of the cathode
6. THE CORNER EFFECT
7. CONCLUSIONS

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1. BASIC FACTS

It is known that electrodeposited metals can have different morphologies at different positions of the electrode surface. This means that the local current density during electrodeposition of metals varies from point to point on the electrode surface. Even for a simple electrode configuration, the calculation of the current distribution in a classical manner is a complex problem, which becomes more complicate with an increasing complexity of the geometry, especially if the limiting diffusion current density varies over the electrode due to different geometric and hydrodynamic conditions.¹⁻⁴

Recently, a method for the determination of the current density distribution based on a simple equation of the electrode kinetics was developed in a manner presented in the literature.⁵⁻¹⁷ The aim of this work is to review them.

2. A STATEMENT OF THE PROBLEM

The current density distribution in electrochemical cells depends on:

- the geometry of the system,
- the conductivity of both the solution and the electrodes,
- the polarization characteristics of electrodes and
- the hydrodynamics of the system.

In this review, the effects of the geometry of the system, the conductivity of the solution and electrodes and the polarization characteristics of electrodes will be considered. It will be assumed that the diffusion layer thickness is the same over all electrode surfaces.

The simplest representation of an electrochemical cell is presented in Fig. 1.

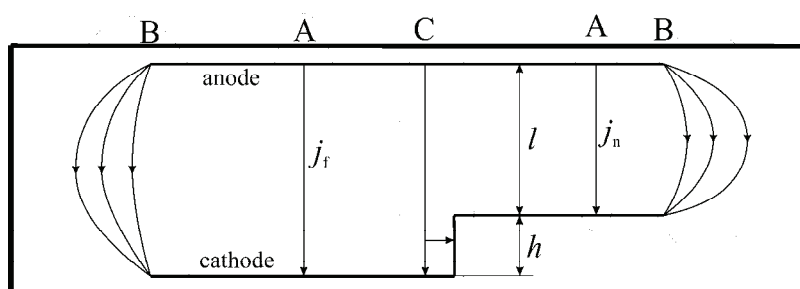


Fig. 1. Schematic representation of an electrochemical cell.

If electrodes are placed in the cell as shown in Fig. 1, three ways for the propagation of the current lines between the anode and cathode are possible.

- there is only one current line between two symmetrically positioned points on the cathode and anode in the homogenous field (A in Fig. 1),
- there are many current lines between two symmetrically positioned points at the edges of both the cathode and anode (B in Fig. 1),

III. there is a division of a current line into two parts at the corner of the cathode (C in Fig. 1).

All these cases will be treated separately.

3. THE CURRENT DENSITY DISTRIBUTION IN HOMOGENOUS FIELDS

The current density distribution between parts of the electrode surface with smaller and larger distance between cathode and anode can be calculated as follows.

The voltage, U , imposed on a cell shown in Fig. 1 is given by:

$$U - E = b_a \log \frac{j_f}{j_{0,a}} + b_c \log \frac{j_f}{j_{0,c}} \frac{j_L}{j_L - j_f} + \rho(l+h)j_f \quad (1)$$

as well as by:

$$U - E = b_a \log \frac{j_n}{j_{0,a}} + b_c \log \frac{j_n}{j_{0,c}} \frac{j_L}{j_L - j_n} + \rho l j_n \quad (2)$$

being valid within the Tafel region and at larger overpotentials, where E is a equilibrium potential difference, b_a and b_c and $j_{0,a}$ and $j_{0,c}$ are the anodic and cathodic Tafel slopes and exchange current densities, respectively, j_L is the limiting diffusion current density for the cathodic process, ρ is the specific ohmic resistivity of the solution and the current densities for the part of the cathode at larger, j_f and smaller, j_n distances from the anode, respectively. For soluble anode considered in this case, it will be $j_{0,a} = j_{0,c} = j_0$ and $E = 0$. In these calculations, absolute values of both the cathodic and anodic current densities, as well as the cathodic and anodic overpotentials are taken. The meanings of l and h are seen from Fig. 1. It is obvious from Eqs. (1) and (2) that:

$$j_n = j_f \quad (3)$$

if $\rho = 0$ and if the system is under complete diffusion control, *i.e.*, if

$$j_n = j_f = j_L \quad (4)$$

It follows from Eq. (3) that the dominant effect on current distribution is the effect of the resistivity of the solution.

On the other hand, if the electrochemical part of Eqs. (1) and (2) can be neglected relative to the ohmic voltage drop, complete ohmic control appears and the relation:

$$j_n = j_f \frac{l+h}{l} = j_f \left(1 + \frac{h}{l}\right) \quad (5)$$

is valid, corresponding to the primary current distribution.

An estimation of the effect of different parameters on the current density distribution can be made from Fig. 2, which shows the dependences of the current

densities at the closer, j_n , and further, j_f , part of the cathode from the anode on the cell voltage, U , for different solution resistivities.¹³

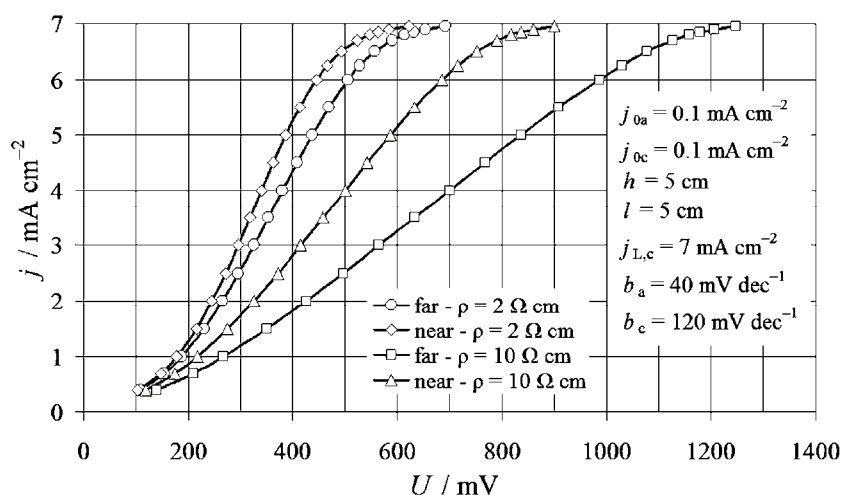


Fig. 2. The dependences of the current densities at parts of the cathode closer to and further from the anode on the cell voltage for different solution resistivities.

As can be seen, the larger is the conductivity of the electrolyte, the more uniform is the current density distribution. A similar but less pronounced effect of the increase of the cathodic Tafel slope can be seen, while the change of j_0 does not affect the current density distribution. It is necessary to note that a soluble anode is considered in this case and, hence, the anodic and cathodic exchange current densities are the same.¹³

Finally, the effect of the h/l ratio will be discussed. As expected, for $h/l \rightarrow 0$, the current density distribution approaches to a uniform one, while for $h/l \gg 1$, it is similar to the primary current distribution. Based on the above results, it follows that the ohmic resistance of the solution is the most important parameter producing an effect on the current density distribution in electrochemical cells. If $h/l \gg 1$, the effect of the geometry of the system can also be important, but an auxiliary anode adjusted to the cathode shape should be used in this case, as illustrated in Fig. 3.

4. THE EDGE EFFECT

The current density distribution in a cell with plane parallel electrodes, with edges not touching the cell wall, is illustrated in Fig. 4.

It can be assumed that a homogenous electric field and, consequently, a uniform current distribution are present over the entire electrode surface up to the very edge of the electrode, where the current density increases abruptly. This problem has been studied in detail in numerous references^{8-10,12,17} and a rela-

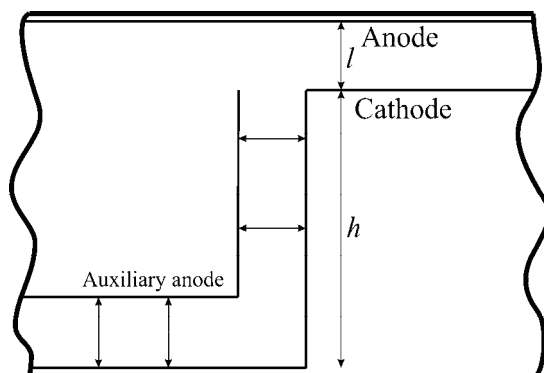


Fig. 3. The auxiliary anode adjusted to the cathode shape in order to improve the current density distribution.

tively rough approximation will be used here. There is only one current line between two symmetrically positioned points on the anode and cathode in a homogenous field, and current density in the homogenous field should be lower than current density at the edges.

There is an infinitely large number of the current lines between two symmetrically placed points at the edges of the electrodes. Hence, it can be taken that the overall resistance between these two points will be equal to an infinitely large number of resistances connected in parallel, being lower than in the homogeneous field. This approximation is more appropriate for the tip of a stationary wire electrode due to the dissipation occurring through the space in this case.¹⁰ On the other hand, in the case of the edges of plane parallel electrodes, it occurs in one plane normal to the electrodes to which the two symmetrically positioned points belong.

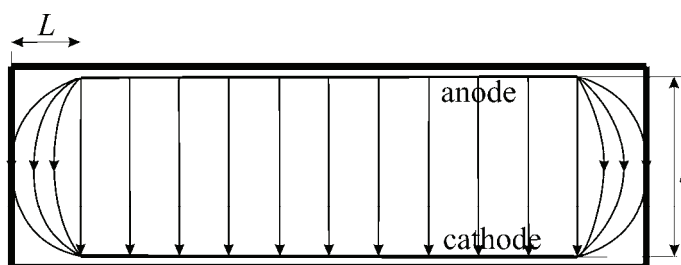


Fig. 4. The current density distribution in a cell with planparallel electrodes the edges of which edges do not touch the cell walls.

5. TWO EQUAL PLANE PARALLEL ELECTRODES ARRANGEMENT

The cell with two equal plane parallel electrodes represents the elementary cell of the electrode arrangement in electrochemical metal refining and winning processes.

It is a well-known fact that in a cell with parallel electrodes (if the electrode edges do not touch the side walls of the cell), the current density is higher at the

edges than at the centre of the electrode. This is because the current flow passes partially around the rectangular space between the electrodes. The increased current density at the edges of the electrodes can be easily noticed by observing the quality of the metal electrodeposit at the cathode. In some cases, the deposit in the central part of the cathode may be compact and flat, whereas the formation of dendrites is observed at the edges. The appearance of dendrites at the edges of cathodes in such situations is the most important problem of the current density distribution, because the growing dendrites could cause short circuits followed by a decrease in the current efficiency, or even damage to the power supply.

The aim of this section is to show in which way dendritic growth at the cathode edges can be avoided in electrowinning and refining processes.

5.1 Ohmic resistance of the cell

The current density distribution in a rectangular electrolytic cell in which parallel electrodes cover only part of the wall and the linear approximation of the current distribution are presented schematically in Fig. 5.

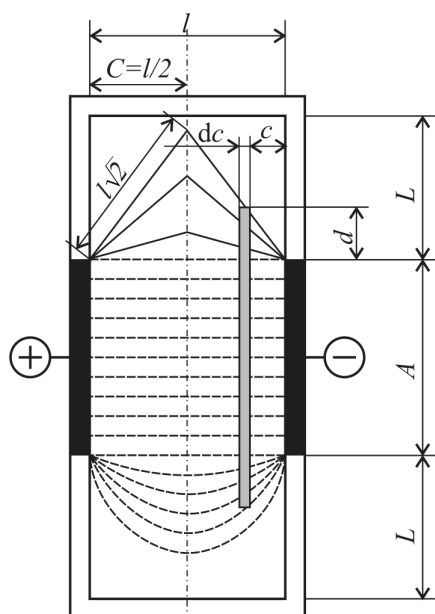


Fig. 5. Schematic presentation of the current distribution in a parallel plate electrode geometry and the linear approximation model showing the current flow passing the space between the plane parallel electrodes (A is the electrode width, L is the distance between the edge of the electrode and the side walls and l is the distance between the electrodes).^{5,9}

The analysis performed here for the current distribution between the electrode edges and the cell side walls is obviously valid also for the situation in which there are gaps between the upper edges of the electrodes and the free surface of the solution and the lower edges to the bottom of the cell. In the case under consideration, these two distances are zero.

The resistance dR of a section of the electrolyte of thickness dc is given by:

$$dR = \frac{\rho}{B} \frac{dc}{A + 2d} \quad (6)$$

where B is the height of the electrode and ρ is the specific resistance of the electrolyte. From the linear approximation, it follows:⁵

$$d = \frac{L}{C}c \quad (7)$$

The parameters d and c are indicated in Fig. 5.

The resistance of the whole electrolyte is then given by:⁵

$$R = \frac{\rho C}{BL} \ln \left(\frac{A + 2L}{A} \right) \quad (8)$$

and for $L \rightarrow 0$, by:

$$\lim_{L \rightarrow 0} R = \frac{2\rho C}{BA} = \frac{\rho l}{BA} = R_h \quad (9)$$

where R_h corresponds to the resistance of a system with a homogeneous current density distribution (the side walls touch the edges of the electrodes). For $0 \leq L \ll \ll \infty$, L can be related to A by a linear coefficient k as follows:

$$L = kA \quad (10)$$

which transforms Eq. (8) to:

$$R = \frac{R_h}{2k} \ln(1 + 2k) \quad (11)$$

and

$$l_{\text{eff}} = \frac{l}{2k} \ln(1 + 2k) \quad (12)$$

taking into account Eq. (9), where l_{eff} represents the interelectrode distance in a cell with $L=0$, the resistance of which is equal to the resistance of a cell in which the interelectrode distance is l and $L > 0$.

The good agreement between the experimental results and the values predicted by Eq. (11) extends to $k \approx 1$. It can be concluded that for this system, Eq. (11) is valid for $k < 1$. This means that the maximum penetration of the current lines occurs when $L = C = A$ in this case, and that the maximum length of the current line, l' , is $l\sqrt{2}$.

5.2 The very edge ohmic resistance

This consideration of the very edge current density can be elaborated mathematically in the following way.⁹ Assuming total ohmic control, the voltage drop in the solution between the electrodes inside the homogenous field is given by:

$$U - E = \rho l j \quad (13)$$

and outside of the homogeneous field by:

$$U - E = \rho l_i j_i \quad (14)$$

where U is the cell voltage, E is the equilibrium potential difference, ρ is the specific resistivity of the electrolyte, l the interelectrode distance, j is the current density, l_i is the length of the i -th current line and j_i is the current density corresponding to the i -th current line, as can be seen from Fig. 6.

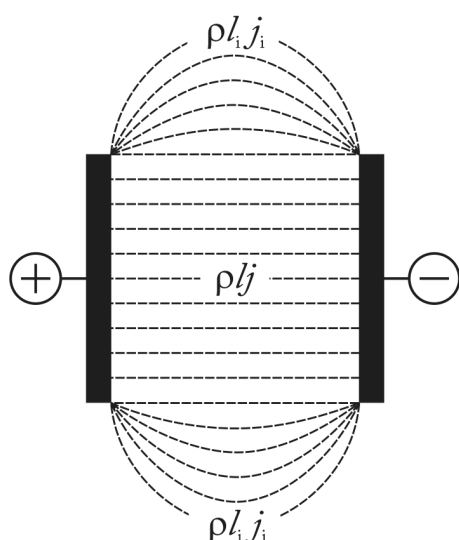


Fig. 6. Current lines between electrodes with edges not touching the side walls of the cell.⁹

The difference in the current lines outside of the homogeneous field is given by:

$$j_i + \Delta j_i - j_i = \frac{U - E}{\rho} \left(\frac{1}{l_i} - \frac{1}{l_i + \Delta l_i} \right) \quad (15)$$

or in the differential form:

$$\frac{d j_i}{d l_i} = \frac{U - E}{\rho} \frac{1}{l_i^2} \quad (16)$$

When Eq. (16) is integrated from the interelectrode distance l to the maximum length of the current line l' , the maximum contribution to the edge current density j' , due to current line propagation between the electrode edges and the side walls of the cell, is obtained:

$$j' = \frac{U - E}{\rho} \left(\frac{1}{l} - \frac{1}{l'} \right) = \frac{U - E}{\rho l} \frac{l' - l}{l'} \quad (17)$$

Taking into accounts Eq. (13), one obtains:

$$j' = j \frac{l' - l}{l'} \quad (18)$$

The edge current density, j_e , can be written as:

$$j_e = j + j' \quad (19)$$

The maximum value of j' is obtained from Eq. (18) as:

$$j'_{\max} = j \frac{2 - \sqrt{2}}{2} \quad (20)$$

Combining Eqs. (19) and (20), the maximum edge current density can be given as:¹¹

$$j_{e, \max} = \left(2 - \frac{\sqrt{2}}{2} \right) j \approx 1.3 j \quad (21)$$

for $l' = l\sqrt{2}$, as follows from Fig. 1.

This means that the very edge resistance is lower than that in the homogenous field and that the minimum effective interelectrode distance, $l_{\text{eff},e,\min}$ between the edges of the anode and cathode will be:

$$l_{\text{eff},e,\min} = \frac{2}{4 - \sqrt{2}} l \quad (22)$$

because:

$$\rho l j = \rho l_{\text{eff},e,\min} j_{e,\max} \quad (23)$$

5.3 The edge effect on the current density distribution

In a cell with parallel plate electrodes, if the electrode edges do not touch the cell side walls, the potential difference between two points in the homogenous field symmetrically positioned on the electrodes is given by:

$$U = E + \eta_a + \eta_c + \rho l j \quad (24)$$

Analogously, the cell voltage at the edges can be expressed as:

$$U = E + \eta_{a,e} + \eta_{c,e} + \rho l_{\text{eff},e} j_e \quad (25)$$

where η_a and η_c are the anodic and cathodic overpotentials corresponding to the homogenous field, respectively, and $\eta_{a,e}$ and $\eta_{c,e}$ are the anodic and cathodic overpotentials corresponding to the edges, respectively.

Elimination of U from Eqs. (24) and (25) gives:

$$\eta_{a,e} + \eta_{c,e} = \eta_a + \eta_c + \rho l j - \rho l_{\text{eff},e} j_e \quad (26)$$

In this case $\rho l j > \rho l_{\text{eff},e} j_e$, because the increase of the current density also leads to the increase of the cathodic and anodic overpotentials.

In this way, a part of the ohmic potential drop in a homogenous field transforms into electrochemical overpotential for points at the plane electrode edges, or in a similar position, meaning the edge current density is larger than in the homogenous field. In this way it is possible to explain the change in the quality of the metal deposit near the edge and at the very edge of an electrode. It should be noted, however, that according to the proposed model, the entire edge current is located at the very edge of the electrode. In other words, a homogeneous electric field and, consequently, a uniform current distribution is assumed over the entire electrode surface up to the very edge of the electrode, where the current density increases abruptly, which is quite close to the real state described by other authors.^{2,18}

5.4 The depth of the penetration of a current line between the electrode edges and the cell side walls

Equation (17) can be rewritten in the form:

$$l' = \frac{l}{1 - \frac{\rho l j'}{U - E}} \quad (27)$$

and if j' is replaced with $j(2 - \sqrt{2})/2$, then l' becomes:

$$l' = \frac{l}{1 - \frac{\rho l j}{U - E} \frac{2 - \sqrt{2}}{2}} \quad (28)$$

as the maximum length of a current line. $(U - E)$ in Eqs. (27) and (28) is the ohmic potential drop, but it can be substituted by the cell potential due to the following facts.

The current along each line should be very low and because of this, the electrochemical overpotentials at the edges of electrodes due to one current line can be neglected relative to the ohmic potential drop. Hence, the cell potential transforms into the ohmic potential drop along each current line and $(U - E)$ in Eq. (28) can be substituted by the cell potential from Eq. (24).

Substitution of $(U - E)$ from Eq. (24) in Eq. (28) gives:

$$l' = \frac{l}{1 - \frac{\rho l j}{\eta_a + \eta_c + \rho l j} \frac{2 - \sqrt{2}}{2}} \quad (29)$$

or after rearrangement:

$$l' = l \frac{\eta_a + \eta_c + \rho l j}{\eta_a + \eta_c + \frac{\sqrt{2}}{2} \rho l j} \quad (30)$$

Assuming a linear approximation of the propagation of a current line, the relation between L' , the maximum depth of the propagation of a current line penetration into the space between the edges of the electrodes and the cell side walls, l and l' , is given by:

$$L' = \frac{1}{2} \sqrt{l'^2 - l^2} \quad (31)$$

Substituting l' from Eq. (30) into Eq. (31) and rearranging gives:

$$L' = \frac{l}{2} \left[\left(\frac{\eta_a + \eta_c + \rho l j}{\eta_a + \eta_c + \frac{\sqrt{2}}{2} \rho l j} \right)^2 - 1 \right]^{1/2} \quad (32)$$

It can be shown that if:

$$\eta_a + \eta_c \gg \rho l j \quad (33)$$

then $l' \rightarrow l$ and $L' \rightarrow 0$ and, in the opposite case, $l' = l\sqrt{2}$ and $L' \rightarrow l/2$. This shows that the ability of an electrolyte to distribute the current density uniformly increases with decreasing $\rho l j$ product, *i.e.*, with decreasing ohmic polarization. Furthermore, it is to be expected that with a larger spacing (the distance between the polarization $j - U$ curves for $L = 0$ and $L > 0$), the current density distribution will become worse.

5.5 The dendritic growth initiation at the edges of the cathode

The equation for the polarization curve is given by:¹⁷

$$j = \frac{j_0 f_c}{1 + \frac{j_0 f_c}{j_L}} \quad (34)$$

for $f_c \gg f_a$, where $f_c = 10^{\eta/b_c}$ and $f_a = 10^{-\eta/b_a}$. The critical overpotential for dendritic growth initiation, η_i , is given by:

$$\eta_i = \frac{b_c}{2.3} \ln \frac{j_L}{j_0} \quad (35)$$

Substitution of η_i from Eq. (35) into Eq. (34) and further rearranging gives:

$$j_i = \frac{1}{2} j_L \quad (36)$$

where j_i is the critical current density for dendritic growth initiation.

If $L > L'$, the edge current density could be obtained by combining Eqs. (18), (19) and (30) as:

$$j_e = j \frac{\eta_a + \eta_e + \left(2 - \frac{\sqrt{2}}{2}\right) \rho l j}{\eta_a + \eta_e + \rho l j} \quad (37)$$

Assuming that the maximum edge current density at which dendrites do not grow is given by Eq. (36), the maximum current density, j_{\max} , in the homogeneous field at which dendrites at the edges do not grow is obtained after the substitution of j_e in Eq. (37) with j_i and after combination of Eqs. (36) and (37) as:

$$j_{\max} = \frac{1}{2} \frac{\eta_a + \eta_e + \rho l j}{\eta_a + \eta_e + \left(2 - \frac{\sqrt{2}}{2}\right) \rho l j} j_L \quad (38)$$

It follows from Eq. (38) that $j_{\max} \approx 0.5 j_L$ for:

$$\eta_a + \eta_c \gg \rho l j \quad (39)$$

and $j_{\max} \approx 0.4 j_L$ for:

$$\eta_a + \eta_c \ll \rho l j \quad (40)$$

The maximum current density in both cases is larger than the current density corresponding to the end of the Tafel linearity, which is the optimum current density for the deposition of compact metal. Hence, if the deposition current density corresponds to the end of the Tafel linearity, dendrites will not grow at the edges of the electrode. It should be noted that in metal electrorefining, the working current density can be determined relative to the initial concentration of depositing ions, because it remains constant or increases during the refining process. In electrowinning processes, the working current density must be determined relative to the final concentration of depositing ions, because it is lower than initial one. The same reason is valid in the case of $L < L'$; meaning, in general, that if the current density in the cell is lower than $0.4 j_L$, dendrites and probably carrot-like protrusion on the electrode edges do not grow.

Equations (39) and (40) are in qualitative agreement with experimental findings, but this topic requires some additional, carefully performed investigations.

6. THE CORNER EFFECT

“Corner weakness” occurs in heavy deposits of electroformed metal at screened cathode parts, *i.e.*, corners. At these areas, the deposit is thinner and, in extreme cases, there is no deposition at all along the line of the corner bisector. A number of microphotographs of deposit cross-sections, illustrating the “corner weakness” effect can be found in the literature.^{19,20} It can be seen that the cal-

culated deposit profile,¹² with a crack appearing along the corner bisector, looks very similar to that typically obtained in plating practice.^{19,20} The consequence is the emergence of a fracture under negligible load along the line of the corner bisection, instead of a fracture at much higher loads across the narrowest cross-section of an electroformed deposit normal to the line of pull.

A theoretical analysis of this phenomenon has been reported using the following assumptions:

- the potential difference between each two points on the anode and cathode is equal to the cell voltage,
- the current lines are normal to the electrode surface,
- the corresponding ohmic resistance of a solution exists along each current line and the current lines are independent and insulated from each other,
- current lines in the vicinity of a protrusion divide into components which are normal to the electrode surface and
- the Kirchoff Laws are valid for current lines branching.

The current distribution in a cell with the electrode arrangement given in Fig. 1 near to an elevation at the cathode can be envisaged as shown in Fig. 7.

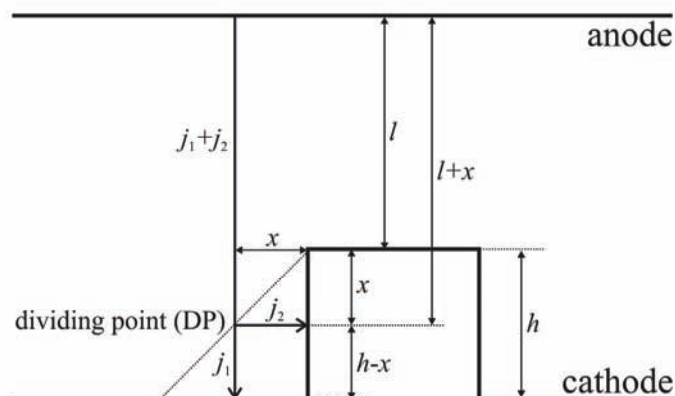


Fig. 7. The assumed model of the line division of the current at the corner on the cathode surface.¹²

According to the assumed model of current line division, it follows that there is no deposition along the line of bisection (Fig. 7). If division of current lines occurs along the line indicated by the dashed line, this is in perfect agreement with some earlier experimental findings.¹² It can be seen that this configuration provides the same density of current lines at the cathode as at the anode.

The overall current density along the current line from the anode to the dividing point (DP) is obviously the sum of the partial ones branching at the DP, *i.e.*, $(j_1 + j_2)$. For a cell with a soluble anode, following relations are valid:

$$U_{\text{cell}} = \frac{b_a}{2.3} \ln \frac{j_1 + j_2}{j_0} + \rho(l+x)(j_1 + j_2) + \frac{b_c}{2.3} \ln \frac{j_1 j_L}{j_0(j_L - j_1)} + \rho j_1(h-x) \quad (41)$$

$$U_{\text{cell}} = \frac{b_a}{2.3} \ln \frac{j_1 + j_2}{j_0} + \rho(l+x)(j_1 + j_2) + \frac{b_c}{2.3} \ln \frac{j_2 j_L}{j_0(j_L - j_2)} + \rho j_2 x \quad (42)$$

The proposed model implies that there is no current component in the direction of the corner vertex and the appearance of a crack along the corner bisector is expected.

A deposit cannot be obtained in the corner of the cathode directly but rather by the buildup of the deposit in the x and y direction. An overlap of the x and y oriented deposits should occur when current density virtually does not depend on the distance from the very corner.

However, if the current density decreases upon approaching the corner vertex, the deposits would not overlap and a flaw would be created.¹²

An improved procedure for the determination of j_1 and j_2 as compared to the one presented earlier¹² is presented here. It follows from Eqs. (41) and (42) that:

$$E_{\text{DPC}} = b_c \log \frac{j_1 j_L}{j_0(j_L - j_1)} + \rho j_1(l-x) \quad (43)$$

and

$$E_{\text{DPC}} = b_c \log \frac{j_2 j_L}{j_0(j_L - j_2)} + \rho j_2 x \quad (44)$$

E_{DPC} should be calculated using Eq. (43) for $x = 0.1h, 0.5h$ and $0.9h$ and $j_1 = 1, 2, 3, 4, 5$ and 6 mA/cm^2 and $\rho = 5 \text{ } \Omega \text{ cm}$, $j_0 = 0.1 \text{ mA cm}^{-2}$, $l = 5 \text{ cm}$, $h = 5 \text{ cm}$, $j_L = 7 \text{ mA cm}^{-2}$, $b_a = 40 \text{ mV dec}^{-1}$ and $b_c = 120 \text{ mV dec}^{-1}$. Then, j_2 can be calculated using Eq. (44) and U is obtained as:

$$U = b_a \log \frac{j_1 + j_2}{j_0} + (l+x)(j_1 + j_2) + E_{\text{DPC}} \quad (45)$$

The calculation should be performed for each x and the obtained values plotted as shown in Figs. 8–10. Taking the calculated values of the current densities, it is possible to visualize the current density distribution in the electrochemical cell schematically presented in Fig. 11. Taking that j_f corresponds to a distance of 1 cm, the j_n, j_f, j_1 and j_2 values for each x can be calculated and the current density distribution can be presented as in Fig. 11 for different values of U , being in agreement with experimental data.^{19,20}

The current density distribution effect is of a high technological significance for the creation of open porous metal structures, denoted as honeycomb-like ones,

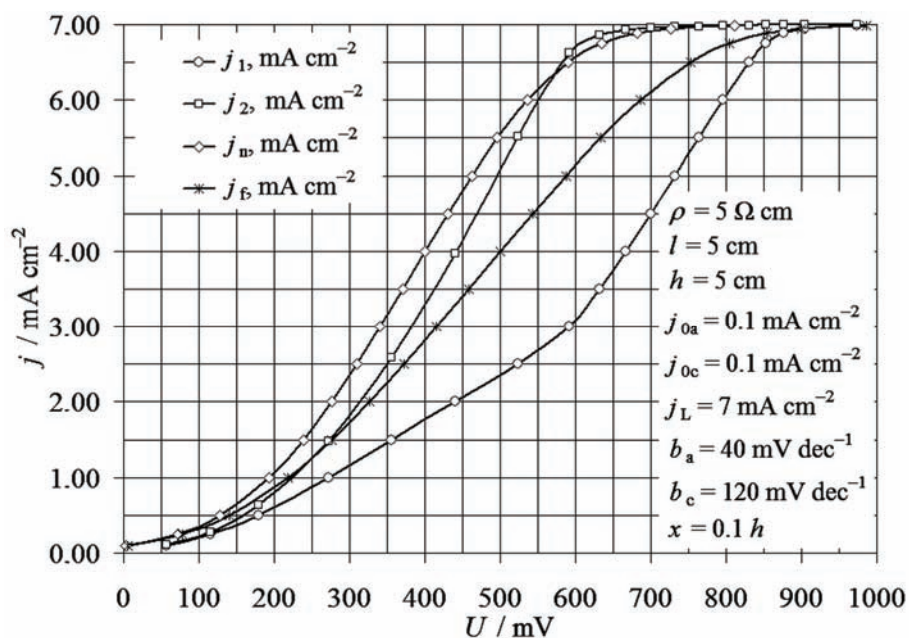


Fig. 8. The dependences of the current densities at the near and at far parts of an electrode on the cell voltage, as well as the j_1 and j_2 for $x = 0.1$.

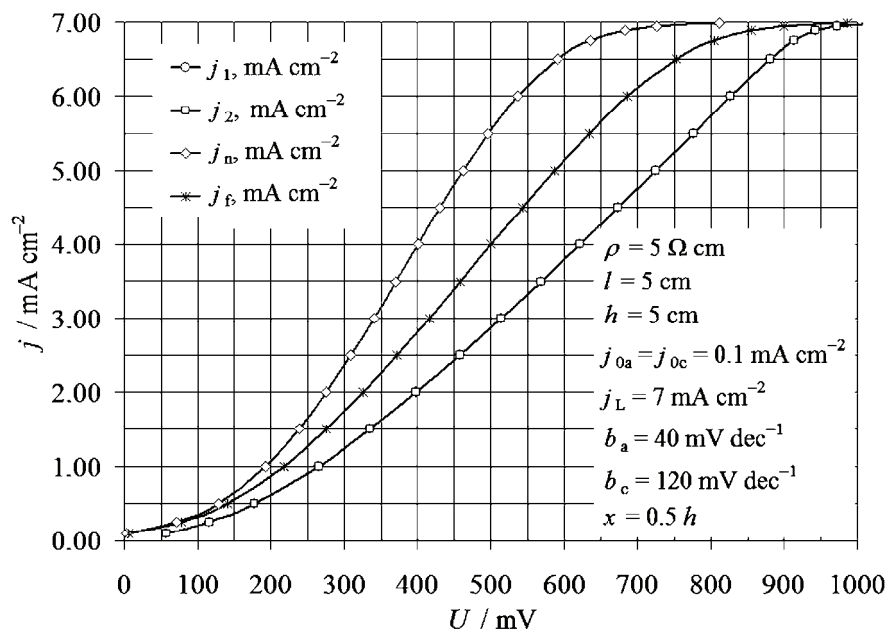


Fig. 9. The dependences of the current densities at the near and at far parts of an electrode on the cell voltage, as well as the j_1 and j_2 for $x = 0.5$.

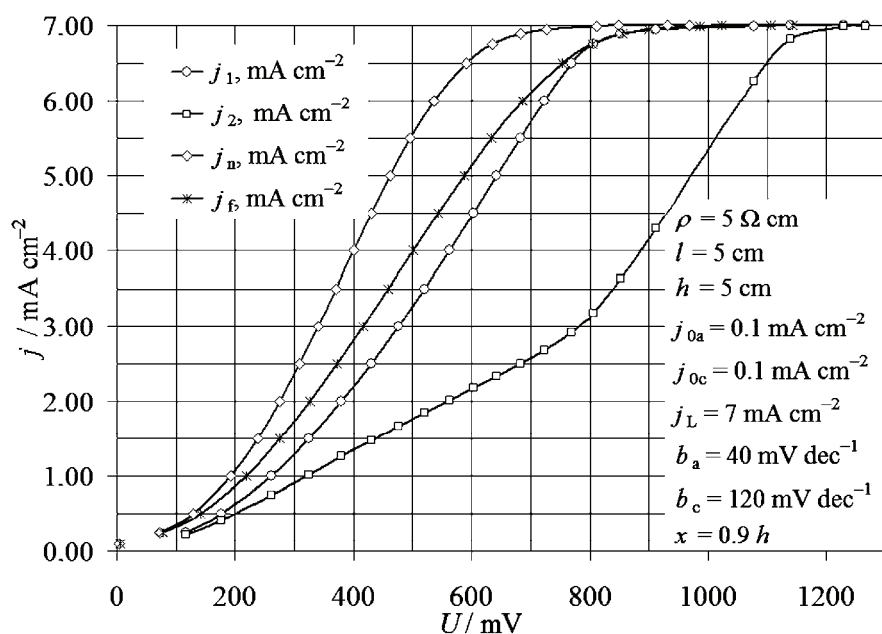


Fig. 10. The dependences of the current densities at the near and at far part of an electrode on the cell voltage, as well as the j_1 and j_2 for $x = 0.9$.

which are ideally suited for electrodes in many electrochemical devices, such as fuel cells, sensors and batteries.²¹ Copper structures of this type are formed from acid sulfate solutions of different CuSO_4 and H_2SO_4 concentrations,²² characterized by different values of the exchange current density, j_0 .²³ Due to the effect of the current density distribution effect, improvement of the micro- and nanostructural characteristics of the honeycomb-like structures can be attained by application of periodically changing regimes of electrolysis, such as the pulsating overpotential (PO) regime.²⁴

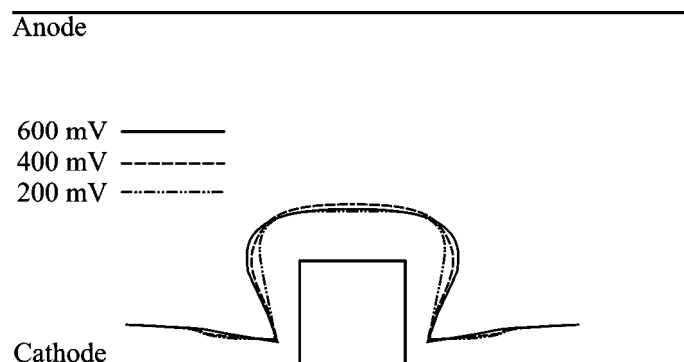


Fig. 11. Visualization of current density distribution for $U = 200, 400$ and 600 mV.

7. CONCLUSIONS

A concise review of earlier results and an improved method for the estimation of the current density distribution in electrochemical cells is presented. The method is based on the simple equations of electrode kinetics and the physical essence of the phenomena is clearly demonstrated.

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ИЗВОД

МАТЕМАТИЧКИ МОДЕЛ РАСПОДЕЛЕ ГУСТИНЕ СТРУЈЕ У
ЕЛЕКТРОХЕМИЈСКИМ ЋЕЛИЈАМА

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Приказан је сажети преглед ранијих резултата и представљен је побољшан приступ процени расподеле струје у електрохемијским ћелијама. Овај приступ, заснован на једначинама електрохемијске кинетике, употребљен је за теоријско објашњење ивичних и угаоних ефеката. Такође су дискутовани утицаји геометрије система, кинетичких параметара катодних реакција и отпорности раствора. Представљена је процедура за комплетну анализу расподеле струје у електрохемијским ћелијама.

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